through the closed surface is

$$\Phi_E = k_e q \oint \frac{dA \cos\theta}{r^2} = k_e q \oint d\Omega = 4\pi k_e q = \frac{q}{\epsilon_0}$$

Thus we have derived Gauss's law, Equation 24.6. Note that this result is independent of the shape of the closed surface and independent of the position of the charge within the surface.

SUMMARY

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A, the electric flux through the surface is

$$\Phi_E = EA\cos\theta \qquad \textbf{(24.2)}$$

In general, the electric flux through a surface is

$$\Phi_E = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} \qquad \textbf{(24.3)}$$

You need to be able to apply Equations 24.2 and 24.3 in a variety of situations, particularly those in which symmetry simplifies the calculation.

Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the *net* charge inside the surface divided by ϵ_0:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \qquad \textbf{(24.6)}$$

Using Gauss's law, you can calculate the electric field due to various symmetric charge distributions. Table 24.1 lists some typical results.

TABLE 24.1 Typical Electric Field Calculations Using Gauss's Law

Charge Distribution	Electric Field	Location
Insulating sphere of radius R, uniform charge density, and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	$k_e \frac{Q}{R^3} r$	$r < R$
Thin spherical shell of radius R and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	0	$r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \frac{\lambda}{r}$	Outside the line
Nonconducting, infinite charged plane having surface charge density σ	$\frac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\frac{\sigma}{\epsilon_0}$	Just outside the conductor
	0	Inside the conductor