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**SUMMARY**

After a particle moves along the  $x$  axis from some initial position  $x_i$  to some final position  $x_f$ , its **displacement** is

$$\Delta x \equiv x_f - x_i \quad (2.1)$$

The **average velocity** of a particle during some time interval is the displacement  $\Delta x$  divided by the time interval  $\Delta t$  during which that displacement occurred:

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t} \quad (2.2)$$

The **average speed** of a particle is equal to the ratio of the total distance it travels to the total time it takes to travel that distance.

The **instantaneous velocity** of a particle is defined as the limit of the ratio  $\Delta x/\Delta t$  as  $\Delta t$  approaches zero. By definition, this limit equals the derivative of  $x$  with respect to  $t$ , or the time rate of change of the position:

$$v_x \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (2.4)$$

The **instantaneous speed** of a particle is equal to the magnitude of its velocity.

The **average acceleration** of a particle is defined as the ratio of the change in its velocity  $\Delta v_x$  divided by the time interval  $\Delta t$  during which that change occurred:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad (2.5)$$

The **instantaneous acceleration** is equal to the limit of the ratio  $\Delta v_x/\Delta t$  as  $\Delta t$  approaches 0. By definition, this limit equals the derivative of  $v_x$  with respect to  $t$ , or the time rate of change of the velocity:

$$a_x \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad (2.6)$$

The **equations of kinematics** for a particle moving along the  $x$  axis with uniform acceleration  $a_x$  (constant in magnitude and direction) are

$$v_{xf} = v_{xi} + a_x t \quad (2.8)$$

$$x_f - x_i = \bar{v}_x t = \frac{1}{2}(v_{xi} + v_{xf}) t \quad (2.10)$$

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2 \quad (2.11)$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \quad (2.12)$$

You should be able to use these equations and the definitions in this chapter to analyze the motion of any object moving with constant acceleration.

An object falling freely in the presence of the Earth's gravity experiences a free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, then the free-fall acceleration  $g$  is constant over the range of motion, where  $g$  is equal to  $9.80 \text{ m/s}^2$ .

Complicated problems are best approached in an organized manner. You should be able to recall and apply the steps of the GOAL strategy when you need them.