Summary

When a rigid object rotates about a fixed axis, each point of the object moves in a circular path. Lines drawn perpendicularly from the rotation axis to various points in the object all sweep out the same angle θ in any given time interval.

Angles are conveniently measured in **radians**, where one radian is the angle subtended by an arc whose length is equal to the radius, or

$$2\pi \text{ rad} = 360^\circ$$
$$1 \text{ rad} \approx 57.3^\circ.$$

Angular velocity, ω, is defined as the rate of change of angular position:

$$\omega = \frac{\Delta\theta}{\Delta t}. \quad \textbf{(8–2)}$$

All parts of a rigid object rotating about a fixed axis have the same angular velocity at any instant.

Angular acceleration, α, is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}. \quad \textbf{(8–3)}$$

The linear velocity v and acceleration a of a point fixed at a distance r from the axis of rotation are related to ω and α by

$$v = r\omega, \quad \textbf{(8–4)}$$

$$a_{\text{tan}} = r\alpha, \quad \textbf{(8–5)}$$

$$a_{\text{R}} = \omega^2 r, \quad \textbf{(8–6)}$$

where a_{tan} and a_{R} are the tangential and radial (centripetal) components of the linear acceleration, respectively.

The frequency f is related to ω by

$$\omega = 2\pi f, \quad \textbf{(8–7)}$$

and to the period T by

$$T = 1/f. \quad \textbf{(8–8)}$$

The equations describing uniformly accelerated rotational motion (α = constant) have the same form as for uniformly accelerated linear motion:

$$\omega = \omega_0 + \alpha t, \qquad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta, \qquad \overline{\omega} = \frac{\omega + \omega_0}{2}. \quad \textbf{(8–9)}$$

The dynamics of rotation is analogous to the dynamics of linear motion. Force is replaced by **torque** τ, which is defined as the product of force times lever arm (perpendicular distance from the line of action of the force to the axis of rotation):

$$\tau = rF\sin\theta = r_\perp F = rF_\perp. \quad \textbf{(8–10)}$$

Mass is replaced by **moment of inertia** I, which depends not only on the mass of the object, but also on how the mass is distributed about the axis of rotation. Linear acceleration is replaced by angular acceleration. The rotational equivalent of Newton's second law is then

$$\Sigma\tau = I\alpha. \quad \textbf{(8–14)}$$

The **rotational kinetic energy** of an object rotating about a fixed axis with angular velocity ω is

$$\text{KE} = \tfrac{1}{2}I\omega^2. \quad \textbf{(8–15)}$$

For an object both translating and rotating, the total kinetic energy is the sum of the translational kinetic energy of the object's center of mass plus the rotational kinetic energy of the object about its center of mass:

$$\text{KE} = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 \quad \textbf{(8–16)}$$

as long as the rotation axis is fixed in direction.

The **angular momentum** L of an object about a fixed rotation axis is given by

$$L = I\omega. \quad \textbf{(8–18)}$$

Newton's second law, in terms of angular momentum, is

$$\Sigma\tau = \frac{\Delta L}{\Delta t}. \quad \textbf{(8–19)}$$

If the net torque on the object is zero, $\Delta L/\Delta t = 0$, so L = constant. This is the **law of conservation of angular momentum** for a rotating object.

The following Table summarizes angular (or rotational) quantities, comparing them to their translational analogs.

Translation	Rotation	Connection
x	θ	$x = r\theta$
v	ω	$v = r\omega$
a	α	$a = r\alpha$
m	I	$I = \Sigma mr^2$
F	τ	$\tau = rF\sin\theta$
$\text{KE} = \frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	
$p = mv$	$L = I\omega$	
$W = Fd$	$W = \tau\theta$	
$\Sigma F = ma$	$\Sigma\tau = I\alpha$	
$\Sigma F = \frac{\Delta p}{\Delta t}$	$\Sigma\tau = \frac{\Delta L}{\Delta t}$	

Questions

1. A bicycle odometer (which measures distance traveled) is attached near the wheel hub and is designed for 27-inch wheels. What happens if you use it on a bicycle with 24-inch wheels?

2. Suppose a disk rotates at constant angular velocity. Does a point on the rim have radial and/or tangential acceleration? If the disk's angular velocity increases uniformly, does the point have radial and/or tangential acceleration? For which cases would the magnitude of either component of linear acceleration change?

3. Could a nonrigid body be described by a single value of the angular velocity ω? Explain.

4. Can a small force ever exert a greater torque than a larger force? Explain.