The energy stored in the magnetic field of an inductor carrying a current I is

$$U = \tfrac{1}{2}LI^2 \tag{32.12}$$

This energy is the magnetic counterpart to the energy stored in the electric field of a charged capacitor.

The energy density at a point where the magnetic field is B is

$$u_B = \frac{B^2}{2\mu_0} \tag{32.14}$$

The mutual inductance of a system of two coils is given by

$$M_{12} = \frac{N_2\Phi_{12}}{I_1} = M_{21} = \frac{N_1\Phi_{21}}{I_2} = M \tag{32.15}$$

This mutual inductance allows us to relate the induced emf in a coil to the changing source current in a nearby coil using the relationships

$$\mathcal{E}_2 = -M_{12}\frac{dI_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M_{21}\frac{dI_2}{dt} \tag{32.16, 32.17}$$

In an LC circuit that has zero resistance and does not radiate electromagnetically (an idealization), the values of the charge on the capacitor and the current in the circuit vary in time according to the expressions

$$Q = Q_{\text{max}} \cos(\omega t + \phi) \tag{32.21}$$

$$I = \frac{dQ}{dt} = -\omega Q_{\text{max}} \sin(\omega t + \phi) \tag{32.23}$$

where Q_{max} is the maximum charge on the capacitor, ϕ is a phase constant, and ω is the angular frequency of oscillation:

$$\omega = \frac{1}{\sqrt{LC}} \tag{32.22}$$

The energy in an LC circuit continuously transfers between energy stored in the capacitor and energy stored in the inductor. The total energy of the LC circuit at any time t is

$$U = U_C + U_L = \frac{Q_{\text{max}}^2}{2C}\cos^2\omega t + \frac{LI_{\text{max}}^2}{2}\sin^2\omega t \tag{32.26}$$

At $t = 0$, all of the energy is stored in the electric field of the capacitor ($U = Q_{\text{max}}^2/2C$). Eventually, all of this energy is transferred to the inductor ($U = LI_{\text{max}}^2/2$). However, the total energy remains constant because energy transformations are neglected in the ideal LC circuit.

QUESTIONS

1. Why is the induced emf that appears in an inductor called a "counter" or "back" emf?
2. The current in a circuit containing a coil, resistor, and battery reaches a constant value. Does the coil have an inductance? Does the coil affect the value of the current?
3. What parameters affect the inductance of a coil? Does the inductance of a coil depend on the current in the coil?
4. How can a long piece of wire be wound on a spool so that the wire has a negligible self-inductance?
5. A long, fine wire is wound as a solenoid with a self-inductance L. If it is connected across the terminals of a battery, how does the maximum current depend on L?
6. For the series RL circuit shown in Figure Q32.6, can the back emf ever be greater than the battery emf? Explain.