

Equation 31.12 is Gauss's law: **The total electric flux through any closed surface equals the net charge inside that surface divided by  $\epsilon_0$ .** This law relates an electric field to the charge distribution that creates it.

Equation 31.13, which can be considered Gauss's law in magnetism, states that **the net magnetic flux through a closed surface is zero.** That is, the number of magnetic field lines that enter a closed volume must equal the number that leave that volume. This implies that magnetic field lines cannot begin or end at any point. If they did, it would mean that isolated magnetic monopoles existed at those points. The fact that isolated magnetic monopoles have not been observed in nature can be taken as a confirmation of Equation 31.13.

Equation 31.14 is Faraday's law of induction, which describes the creation of an electric field by a changing magnetic flux. This law states that **the emf, which is the line integral of the electric field around any closed path, equals the rate of change of magnetic flux through any surface area bounded by that path.** One consequence of Faraday's law is the current induced in a conducting loop placed in a time-varying magnetic field.

Equation 31.15, usually called the Ampère–Maxwell law, is the generalized form of Ampère's law, which describes the creation of a magnetic field by an electric field and electric currents: **The line integral of the magnetic field around any closed path is the sum of  $\mu_0$  times the net current through that path and  $\epsilon_0\mu_0$  times the rate of change of electric flux through any surface bounded by that path.**

Once the electric and magnetic fields are known at some point in space, the force acting on a particle of charge  $q$  can be calculated from the expression

Lorentz force law

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \quad (31.16)$$

This relationship is called the **Lorentz force law.** (We saw this relationship earlier as Equation 29.16.) Maxwell's equations, together with this force law, completely describe all classical electromagnetic interactions.

It is interesting to note the symmetry of Maxwell's equations. Equations 31.12 and 31.13 are symmetric, apart from the absence of the term for magnetic monopoles in Equation 31.13. Furthermore, Equations 31.14 and 31.15 are symmetric in that the line integrals of  $\mathbf{E}$  and  $\mathbf{B}$  around a closed path are related to the rate of change of magnetic flux and electric flux, respectively. "Maxwell's wonderful equations," as they were called by John R. Pierce,<sup>3</sup> are of fundamental importance not only to electromagnetism but to all of science. Heinrich Hertz once wrote, "One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than we put into them."

## SUMMARY

**Faraday's law of induction** states that the emf induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (31.1)$$

where  $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$  is the magnetic flux.

<sup>3</sup> John R. Pierce, *Electrons and Waves*, New York, Doubleday Science Study Series, 1964. Chapter 6 of this interesting book is recommended as supplemental reading.