What is meant by **curved space**? To understand, recall that our normal method of viewing the world is via Euclidean plane geometry. In Euclidean geometry, there are many axioms and theorems we take for granted, such as that the sum of the angles of any triangle is 180°. Non-Euclidean geometries, which involve curved space, have also been imagined by mathematicians. It is hard enough to imagine three-dimensional curved space, much less curved four-dimensional space-time. So let us try to understand the idea of curved space by using two-dimensional surfaces.

Consider, for example, the two-dimensional surface of a sphere. It is clearly curved, Fig. 33–15, at least to us who view it from the outside—from our three-dimensional world. But how would hypothetical two-dimensional creatures determine whether their two-dimensional space were flat (a plane) or curved? One way would be to measure the sum of the angles of a triangle. If the surface is a plane, the sum of the angles is 180°, as we learn in plane geometry. But if the space is curved, and a sufficiently large triangle is constructed, the sum of the angles will *not* be 180°. To construct a triangle on a curved surface, say the sphere of Fig. 33–15, we must use the equivalent of a straight line: that is, the shortest distance between two points, which is called a **geodesic**. On a sphere, a geodesic is an arc of a great circle (an arc in a plane passing through the center of the sphere) such as the Earth's equator and the Earth's longitude lines. Consider, for example, the large triangle of Fig. 33–15: its sides are two longitude lines passing from the north pole to the equator, and the third side is a section of the equator as shown. The two longitude lines make 90° angles with the equator (look at a world globe to see this more clearly). They make an angle with each other at the north pole, which could be, say, 90° as shown; the sum of these angles is $90° + 90° + 90° = 270°$. This is clearly *not* a Euclidean space. Note, however, that if the triangle is small in comparison to the radius of the sphere, the angles will add up to nearly 180°, and the triangle (and space) will seem flat.

Geodesic

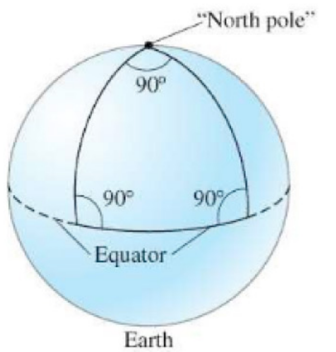


FIGURE 33–15 On a two-dimensional curved surface, the sum of the angles of a triangle may not be 180°.

Another way to test the curvature of space is to measure the radius r and circumference C of a large circle. On a plane surface, $C = 2\pi r$. But on a two-dimensional spherical surface, C is *less* than $2\pi r$, as can be seen in Fig. 33–16. The proportionality between C and r is *less* than 2π. Such a surface is said to have *positive curvature*. On the saddlelike surface of Fig. 33–17, the circumference of a circle is greater than $2\pi r$, and the sum of the angles of a triangle is less than 180°. Such a surface is said to have a *negative curvature*.

FIGURE 33–16 On a spherical surface (a two-dimensional world) a circle of circumference C is drawn about point O as the center. The radius of the circle (not the sphere) is the distance r along the surface. (Note that in our three-dimensional view, we can tell that $2\pi a = C$; since $r > a$, then $2\pi r > C$.)

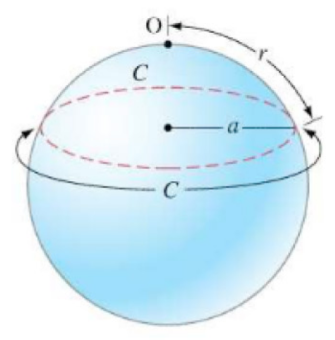
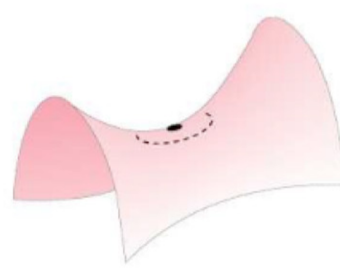


FIGURE 33–17 Example of a two-dimensional surface with negative curvature.



Curvature of the Universe

Now, what about our universe? On a large scale (not just near a large mass), what is the overall curvature of the universe? Does it have positive curvature, negative curvature, or is it flat (zero curvature)?

The universe: open or closed?

If the universe had a positive curvature, the universe would be *closed*, or *finite* in volume. This would *not* mean that the stars and galaxies extended out to a certain boundary, beyond which there is empty space. There is no boundary or edge in such a universe. If a particle were to move in a straight line in a particular direction, it would eventually return to the starting point—perhaps eons of time later.