Since unification occurs at such tiny distances and huge energies, the theory is difficult to test experimentally. But it is not completely impossible. One testable prediction is the idea that the proton might decay (via, for example, $p \rightarrow \pi^0 + e^+$) and violate conservation of baryon number. This could happen if two quarks approached to within 10^{-31} m of each other. But it is very unlikely at normal temperature and energy, so the decay of a proton can only be an unlikely process. In the simplest form of GUT, the theoretical estimate of the proton lifetime for the decay mode $p \rightarrow \pi^0 + e^+$ is about 10^{31} yr, and this has just come within the realm of testability.† Proton decays have still not been seen, and experiments put the lower limit on the proton lifetime for the above mode to be about 10^{33} yr, somewhat greater than this prediction. This may seem a disappointment, but on the other hand, it presents a challenge. Indeed more complex GUTs are not affected by this result.

Proton decay?

EXAMPLE 32–10 ESTIMATE Proton decay. An experiment uses 3300 tons of water waiting to see a proton decay of the type $p \rightarrow \pi^0 + e^+$. If the experiment is run for 4 years without detecting a decay, estimate the lower limit on the proton half-life.

APPROACH As with radioactive decay, the number of decays is proportional to the number of parent species (N), the time interval (Δt), and the decay constant (λ) which is related to the half-life $T_{\frac{1}{2}}$ by (see Eqs. 30–3 and 30–6):

$$\Delta N = -\lambda N \, \Delta t = -\frac{\ln 2}{T_{\frac{1}{2}}} N \, \Delta t.$$

SOLUTION Dealing only with magnitudes, we solve for $T_{\frac{1}{2}}$:

$$T_{\frac{1}{2}} = \frac{N}{\Delta N} \Delta t \ln 2.$$

Thus for $\Delta N < 1$ over the four-year trial,

$$T_{\frac{1}{2}} > N(4\,\text{yr})(0.693),$$

where N is the number of protons in 3300 tons of water. To determine N, we note that each molecule of H_2O contains $(2 + 8 =)10$ protons. So one mole of water (18 g, 6×10^{23} molecules) contains $10 \times 6 \times 10^{23}$ protons in 18 g of water, or about 3×10^{26} protons per kilogram. One ton is 10^3 kg, so the chamber contains $(3.3 \times 10^6\,\text{kg})(3 \times 10^{26}\,\text{protons/kg}) \approx 1 \times 10^{33}$ protons. Then our very rough estimate for a lower limit on the proton half-life is $T_{\frac{1}{2}} > (10^{33})(4\,\text{yr})(0.7) \approx 3 \times 10^{33}$ yr.

An interesting prediction of unified theories relates to cosmology (Chapter 33). It is thought that during the first 10^{-35} s after the theorized Big Bang that created the universe, the temperature was so extremely high that particles had energies corresponding to the unification scale. Baryon number would not have been conserved then, perhaps allowing an imbalance that might account for the observed predominance of matter ($B > 0$) over antimatter ($B < 0$) in the universe.

Connection with cosmology

This last example is interesting, for it illustrates a deep connection between investigations at either end of the size scale: theories about the tiniest objects (elementary particles) have a strong bearing on the understanding of the universe on a large scale. We will look at this more in the next Chapter.

†This is much larger than the age of the universe ($\approx 14 \times 10^9$ yr). But we don't have to wait 10^{31} yr to see. Instead we can wait for one decay among 10^{31} protons over a year (see Eqs. 30–3 and 30–6, $\Delta N = \lambda N \, \Delta t = 0.693 N \, \Delta t / T_{\frac{1}{2}}$).