Cyclotron frequency

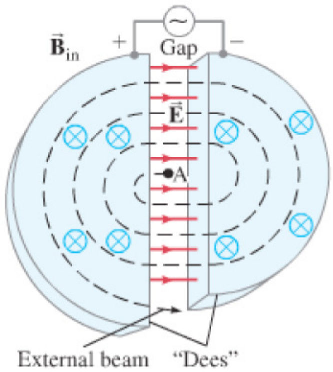

FIGURE 32–2 (repeated) Diagram of a cyclotron.

The frequency, f, of the applied voltage must be equal to that of the circulating protons. When ions of charge q are circulating *within* the hollow dees, the net force F on each is due to the magnetic field B, so $F = qvB$, where v is the speed of the ion at a given moment (Eq. 20–4). The magnetic force is perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ and causes the ions to move in circles; the acceleration inside the dees is thus centripetal and equals v^2/r, where r is the radius of the ion's path at a given moment. We use Newton's second law, $F = ma$, and find that

$$F = ma$$

$$qvB = \frac{mv^2}{r}$$

when the protons are within the dees (not the gap), so

$$v = \frac{qBr}{m}.$$

The time required for a complete revolution is the period T and is equal to

$$T = \frac{\text{distance}}{\text{speed}} = \frac{2\pi r}{qBr/m} = \frac{2\pi m}{qB}.$$

Hence the frequency of revolution f is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}. \qquad \textbf{(32–2)}$$

This is known as the **cyclotron frequency**.

EXAMPLE 32–2 **Cyclotron.** A small cyclotron of maximum radius $R = 0.25\,\text{m}$ accelerates protons in a 1.7-T magnetic field. Calculate (*a*) the frequency needed for the applied alternating voltage, and (*b*) the kinetic energy of protons when they leave the cyclotron.

APPROACH The frequency of the protons revolving within the dees (Eq. 32–2) must equal the frequency of the voltage applied across the gap if the protons are going to increase in speed.

SOLUTION (*a*) From Eq. 32–2,

$$f = \frac{qB}{2\pi m}$$

$$= \frac{(1.6 \times 10^{-19}\,\text{C})(1.7\,\text{T})}{(6.28)(1.67 \times 10^{-27}\,\text{kg})} = 2.6 \times 10^{7}\,\text{Hz} = 26\,\text{MHz},$$

which is in the radio-wave region of the EM spectrum (Fig. 22–8).

(*b*) The protons leave the cyclotron at $r = R = 0.25\,\text{m}$. From $qvB = mv^2/r$ (see above), we have $v = qBr/m$, so

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{q^2B^2R^2}{m^2} = \frac{q^2B^2R^2}{2m}$$

$$= \frac{(1.6 \times 10^{-19}\,\text{C})^2(1.7\,\text{T})^2(0.25\,\text{m})^2}{(2)(1.67 \times 10^{-27}\,\text{kg})} = 1.4 \times 10^{-12}\,\text{J} = 8.7\,\text{MeV}.$$

The KE is much less than the rest energy of the proton (938 MeV), so relativity is not needed.

NOTE The magnitude of the voltage applied to the dees does not affect the final energy. But the higher this voltage, the fewer the revolutions required to bring the protons to full energy.

An important aspect of the cyclotron is that the frequency of the applied voltage, as given by Eq. 32–2, does not depend on the radius r of the particle's path. Thus the frequency does not have to be changed as the protons or ions