

SOLUTION Each electron has $\text{KE} = 1.3 \text{ GeV} = 1300 \text{ MeV}$, which is about 2500 times the rest mass energy of the electron ($0.51 \text{ MeV}/c^2$). Thus we can ignore the term $(m_0c^2)^2$ in Eq. 26-10, $E^2 = p^2c^2 + m_0^2c^4$, and we solve for p :

$$p = \sqrt{\frac{E^2 - m_0^2c^4}{c^2}} \approx \sqrt{\frac{E^2}{c^2}} = \frac{E}{c}.$$

Therefore the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{E},$$

where $E = 1.3 \text{ GeV}$. Hence

$$\lambda = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.0 \times 10^8 \text{ m/s})}{(1.3 \times 10^9 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})} = 0.96 \times 10^{-15} \text{ m},$$

or 0.96 fm. This resolution of about 1 fm is on the order of the size of nuclei (see Eq. 30-1).

NOTE The maximum possible resolution of this beam of electrons is far greater than for a light beam in a light microscope ($\lambda \approx 500 \text{ nm}$).

EXERCISE A What is the wavelength of a proton with $\text{KE} = 1.00 \text{ TeV}$?

Another major reason for building high-energy accelerators is that new particles of greater mass can be produced at higher energies, transforming the KE of the colliding particles into massive particles by $E = mc^2$, as we will discuss shortly. Now we look at particle accelerators.

Cyclotron

The cyclotron was developed in 1930 by E. O. Lawrence (1901–1958; Fig. 32-1) at the University of California, Berkeley. It uses a magnetic field to maintain charged ions—usually protons—in nearly circular paths. Although particle physicists no longer use simple cyclotrons, they are used widely in medicine for treating cancer, and their operating principles are useful for understanding modern accelerators. The protons move in a vacuum inside two D-shaped cavities, as shown in Fig. 32-2. Each time they pass into the gap between the “dees,” a voltage accelerates them (the electric force), increasing their speed and increasing the radius of curvature of their path in the magnetic field. After many revolutions, the protons acquire high kinetic energy and reach the outer edge of the cyclotron where they strike a target. The protons speed up only when they are in the gap between the dees, and the voltage must be alternating. When protons are moving to the right across the gap in Fig. 32-2, the right dee must be electrically negative and the left one positive. A half-cycle later, the protons are moving to the left, so the left dee must be negative in order to accelerate them.



FIGURE 32-1 Ernest O. Lawrence, around 1930, holding the first cyclotron (we see the vacuum chamber enclosing it).

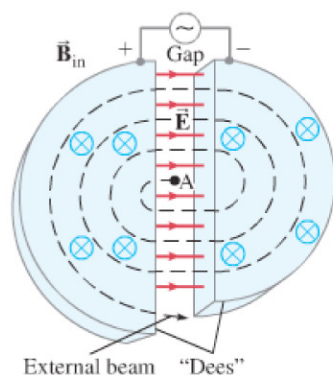


FIGURE 32-2 Diagram of a cyclotron. The magnetic field, applied by a large electromagnet, points into the page. The protons start at A, the ion source. The field lines shown are for the alternating electric field in the gap at a certain moment.