Additional Examples

Here are some more worked-out Examples to give you practice in solving a wide range of Problems.

PHYSICS APPLIED
Elevator (as Atwood's machine)

EXAMPLE 4–13 Elevator and counterweight (Atwood's machine). A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood's machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\text{ kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\text{ kg}$. For the latter case ($m_E = 1150\text{ kg}$), calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

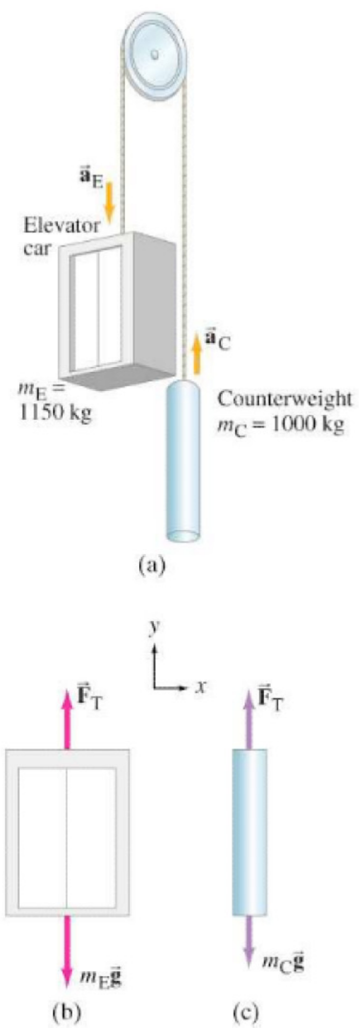

FIGURE 4–23 Example 4–13. (a) Atwood's machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{\mathbf{F}}_T$. Figures 4–23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable doesn't stretch). For the counterweight, $m_C g = (1000\text{ kg})(9.80\text{ m/s}^2) = 9800\text{ N}$, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150\text{ kg})(9.80\text{ m/s}^2) = 11{,}300\text{ N}$, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}g = \frac{1150\text{ kg} - 1000\text{ kg}}{1150\text{ kg} + 1000\text{ kg}}g = 0.070\,g = 0.68\text{ m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\text{ m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations, setting $a = 0.070g = 0.68\text{ m/s}^2$:

$$F_T = m_E g - m_E a = m_E(g - a)$$
$$= 1150\text{ kg}\,(9.80\text{ m/s}^2 - 0.68\text{ m/s}^2) = 10{,}500\text{ N},$$
$$F_T = m_C g + m_C a = m_C(g + a)$$
$$= 1000\text{ kg}\,(9.80\text{ m/s}^2 + 0.68\text{ m/s}^2) = 10{,}500\text{ N},$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

PROBLEM SOLVING
Check your result by seeing if it works in situations where the answer is easily guessed

NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal ($m_E = m_C$), then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass ($m_E \neq 0$) would be predicted by our equation to accelerate at $a = g$, again as expected.