The net effect of this sequence, which is called the **proton–proton cycle**, is for four protons to combine to form one ^4_2He nucleus plus two positrons, two neutrinos, and two gamma rays:

$$4\,^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\text{e}^+ + 2\nu + 2\gamma. \qquad \textbf{(31–7)}$$

Note that it takes two of each of the first two reactions (Eqs. 31–6a and b) to produce the two ^3_2He for the third reaction. So the total energy release for the net reaction, Eq. 31–7, is $(2 \times 0.42\,\text{MeV} + 2 \times 5.49\,\text{MeV} + 12.86\,\text{MeV}) = 24.7\,\text{MeV}$. However, each of the two e^+ (Eq. 31–6a) quickly annihilates with an electron to produce $2m_e c^2 = 1.02\,\text{MeV}$; so the total energy released is $(24.7\,\text{MeV} + 2 \times 1.02\,\text{MeV}) = 26.7\,\text{MeV}$. The first reaction, the formation of deuterium from two protons (Eq. 31–6a), has a very low probability, and the infrequency of that reaction serves to limit the rate at which the Sun produces energy.

EXAMPLE 31–7 ESTIMATE Estimating fusion energy. Estimate the energy released if the following reaction occurred: $^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He}$.

APPROACH We use Fig. 31–11.

SOLUTION We see in Fig. 31–11 that each ^2_1H has a binding energy of about $1\frac{1}{4}\,\text{MeV/nucleon}$, which for 2 nuclei of mass 2 is $4 \times \left(1\frac{1}{4}\right) \approx 5\,\text{MeV}$. The ^4_2He has a binding energy per nucleon of about 7 MeV for a total of $4 \times 7\,\text{MeV} = 28\,\text{MeV}$. Hence the energy release is $28\,\text{MeV} - 5\,\text{MeV} = 23\,\text{MeV}$.

In stars hotter than the Sun, it is more likely that the energy output comes principally from the **carbon** (or **CNO**) **cycle**, which comprises the following sequence of reactions:

$$^{12}_{6}\text{C} + {}^1_1\text{H} \rightarrow {}^{13}_{7}\text{N} + \gamma$$
$$^{13}_{7}\text{N} \rightarrow {}^{13}_{6}\text{C} + \text{e}^+ + \nu$$
$$^{13}_{6}\text{C} + {}^1_1\text{H} \rightarrow {}^{14}_{7}\text{N} + \gamma$$
$$^{14}_{7}\text{N} + {}^1_1\text{H} \rightarrow {}^{15}_{8}\text{O} + \gamma$$
$$^{15}_{8}\text{O} \rightarrow {}^{15}_{7}\text{N} + \text{e}^+ + \nu$$
$$^{15}_{7}\text{N} + {}^1_1\text{H} \rightarrow {}^{12}_{6}\text{C} + {}^4_2\text{He}.$$

Carbon cycle (some stars)

It is easy to see (see Problem 35) that no carbon is consumed in this cycle (see first and last equations) and that the net effect is the same as the proton–proton cycle, Eq. 31–7 (plus 1 extra γ). The theory of the proton–proton cycle and of the carbon cycle as the source of energy for the Sun and stars was first worked out by Hans Bethe (1906–2005) in 1939.

CONCEPTUAL EXAMPLE 31–8 Stellar fusion. What is the heaviest element likely to be produced in fusion processes in stars?

RESPONSE Fusion is possible if the final products have more binding energy (less mass) than the reactants, for then there is net release of energy. Since the binding energy curve in Fig. 31–11 (or Fig. 30–1) peaks near $A \approx 56$ to 58 which corresponds to iron or nickel, it would not be energetically favorable to produce elements heavier than that. Nevertheless, in the center of massive stars or in supernova explosions, there is enough initial kinetic energy available to drive endothermic reactions that produce heavier elements, as well.