

APPROACH Since the neutron and boron are both essentially at rest, the total momentum before the reaction is zero; momentum is conserved and so must be zero afterward as well. Thus,

$$M_{\text{Li}} v_{\text{Li}} = M_{\text{He}} v_{\text{He}}.$$

We solve this for v_{Li} and substitute it into the equation for kinetic energy.

SOLUTION (a) We can use classical KE with little error, rather than relativistic formulas, because $v_{\text{He}} = 9.30 \times 10^6 \text{ m/s}$ is not close to the speed of light c , and v_{Li} will be even less since $M_{\text{Li}} > M_{\text{He}}$. Thus we can write:

$$\text{KE}_{\text{Li}} = \frac{1}{2} M_{\text{Li}} v_{\text{Li}}^2 = \frac{1}{2} M_{\text{Li}} \left(\frac{M_{\text{He}} v_{\text{He}}}{M_{\text{Li}}} \right)^2 = \frac{M_{\text{He}}^2 v_{\text{He}}^2}{2M_{\text{Li}}}.$$

We put in numbers, changing the mass in u to kg and recall that $1.60 \times 10^{-13} \text{ J} = 1 \text{ MeV}$:

$$\begin{aligned} \text{KE}_{\text{Li}} &= \frac{(4.0026 \text{ u})^2 (1.66 \times 10^{-27} \text{ kg/u})^2 (9.30 \times 10^6 \text{ m/s})^2}{2(7.0160 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} \\ &= 1.64 \times 10^{-13} \text{ J} = 1.02 \text{ MeV}. \end{aligned}$$

(b) We are given the data $\text{KE}_a = \text{KE}_x = 0$ in Eq. 31-2, so $Q = \text{KE}_{\text{Li}} + \text{KE}_{\text{He}}$, where

$$\begin{aligned} \text{KE}_{\text{He}} &= \frac{1}{2} M_{\text{He}} v_{\text{He}}^2 = \frac{1}{2} (4.0026 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})(9.30 \times 10^6 \text{ m/s})^2 \\ &= 2.87 \times 10^{-13} \text{ J} = 1.80 \text{ MeV}. \end{aligned}$$

Hence, $Q = 1.02 \text{ MeV} + 1.80 \text{ MeV} = 2.82 \text{ MeV}$.

EXAMPLE 31-3 Will the reaction “go”? Can the reaction $p + {}^{13}_6\text{C} \rightarrow {}^{13}_7\text{N} + n$ occur when ${}^{13}_6\text{C}$ is bombarded by 2.0-MeV protons?

APPROACH The reaction will “go” if the reaction is exothermic ($Q > 0$) and even if $Q < 0$ if the input momentum and kinetic energy are sufficient. First we calculate Q from the difference between final and initial masses using Eq. 31-1, and looking up the masses in Appendix B.

SOLUTION The total masses before and after the reaction are:

Before	After
$M({}^{13}_6\text{C}) = 13.003355$	$M({}^{13}_7\text{N}) = 13.005739$
$M({}^1_1\text{H}) = 1.007825$	$M(n) = 1.008665$
14.011180	14.014404

(We must use the mass of the ${}^1_1\text{H}$ atom rather than that of the bare proton because the masses of ${}^{13}_6\text{C}$ and ${}^{13}_7\text{N}$ include the electrons, and we must include an equal number of electron masses on each side of the equation since none are created or destroyed.) The products have an excess mass of

$$(14.014404 - 14.011180)\text{u} = 0.003224 \text{ u} \times 931.5 \text{ MeV/u} = 3.00 \text{ MeV}.$$

Thus $Q = -3.00 \text{ MeV}$, and the reaction is endothermic. This reaction requires energy, and the 2.0 MeV protons do not have enough to make it go.

NOTE The proton in Example 31-3 would have to have somewhat more than 3.00 MeV of KE to make this reaction go; 3.00 MeV would be enough to conserve energy, but a proton of this energy would produce the ${}^{13}_7\text{N}$ and n with no KE and hence no momentum. Since an incident 3.0-MeV proton has momentum, conservation of momentum would be violated. A calculation using conservation of energy *and* of momentum, as we did in Examples 30-7 and 31-2, shows that the minimum proton energy, called the **threshold energy**, is 3.23 MeV in this case.

Threshold
energy