Additional Example

PHYSICS APPLIED
Archeological dating

EXAMPLE 30–13 **An ancient animal.** The mass of carbon in an animal bone fragment found in an archeological site is 200 g. If the bone registers an activity of 16 decays/s, what is its age?

APPROACH First we determine how many ${}^{14}_{6}C$ atoms there were in our 200-g sample when the animal was alive, given the known fraction of ${}^{14}_{6}C$, 1.3×10^{-12}. Then we use Eq. 30–3b to find the activity back then, and Eq. 30–5 to find out how long ago that was by solving for the time t.

SOLUTION The 200 g of carbon is nearly all ${}^{12}_{6}C$; 12.0 g of ${}^{12}_{6}C$ contains 6.02×10^{23} atoms, so 200 g contains

$$\left(\frac{6.02 \times 10^{23}\text{ atoms}}{12\text{ g}}\right)(200\text{ g}) = 1.00 \times 10^{25}\text{ atoms.}$$

When the animal was alive, the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ in the bone was 1.3×10^{-12}. The number of ${}^{14}_{6}C$ nuclei at that time was

$$N_0 = (1.00 \times 10^{25}\text{ atoms})(1.3 \times 10^{-12}) = 1.3 \times 10^{13}\text{ atoms.}$$

From Eq. 30–3b the magnitude of the activity when the animal was alive ($t = 0$) was

$$\left(\frac{\Delta N}{\Delta t}\right)_0 = \lambda N_0,$$

where $\lambda = 3.83 \times 10^{-12}\text{ s}^{-1}$ as we calculated in Example 30–9. So the original activity was

$$\left(\frac{\Delta N}{\Delta t}\right)_0 = \lambda N_0 = (3.83 \times 10^{-12}\text{ s}^{-1})(1.3 \times 10^{13}) = 50\text{ s}^{-1}.$$

From Eq. 30–5

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\lambda t}$$

where $\Delta N/\Delta t$ is given as 16 s^{-1}. Then

$$16\text{ s}^{-1} = (50\text{ s}^{-1})e^{-\lambda t}$$

or

$$e^{\lambda t} = \frac{50}{16}.$$

We take natural logs of both sides to obtain

$$t = \frac{1}{\lambda}\ln\left(\frac{50}{16}\right) = \frac{1}{3.83 \times 10^{-12}\text{ s}^{-1}}\ln\left(\frac{50}{16}\right)$$

$$= 2.98 \times 10^{11}\text{ s} = 9400\text{ yr},$$

which is the time elapsed since the death of the animal.

Geological Time Scale Dating

PHYSICS APPLIED
Geological dating

Carbon dating is useful only for determining the age of objects less than about 60,000 years old. The amount of ${}^{14}_{6}C$ remaining in objects older than that is usually too small to measure accurately, although new techniques are allowing detection of even smaller amounts of ${}^{14}_{6}C$, pushing the time frame further back. On the other hand, radioactive isotopes with longer half-lives can be used in certain circumstances to obtain the age of older objects. For example, the decay of ${}^{238}_{92}U$, because of its long half-life of 4.5×10^9 years, is useful in determining the ages of rocks on a geologic time scale. When molten material on Earth long ago solidified into rock as the temperature dropped, different compounds solidified according to the melting points, and thus different compounds separated to some extent.