CONCEPTUAL EXAMPLE 30–10 **Safety: activity versus half-life.** One might think that a short half-life material is safer than a long half-life material because it will not last as long. Is this an accurate representation of the situation?

RESPONSE No. A shorter half-life means the activity is higher and thus more "radioactive" and dangerous. On the other hand, a shorter half-life means the material will all decay to a low level sooner. For the same sample size N, a short half-life material is more radioactive but for a shorter time.

EXERCISE D The isotope ${}^{60}_{27}\text{Co}$, used for radiation treatments in hospitals, has a half-life of about 5.3 years. If a hospital buys a ${}^{60}_{27}\text{Co}$ sample, will it all be gone in 10.6 years (two half-lives)?

Additional Example

EXAMPLE 30–11 **A sample of radioactive ${}^{13}_{7}\text{N}$.** A laboratory has 1.49 μg of pure ${}^{13}_{7}\text{N}$, which has a half-life of 10.0 min (600 s). (*a*) How many nuclei are present initially? (*b*) What is the activity initially? (*c*) What is the activity after 1.00 h? (*d*) After approximately how long will the activity drop to less than one per second $(1\ \text{s}^{-1})$?

APPROACH We use the definition of the mole and Avogadro's number (Sections 13–7 and 13–9) to find the number of nuclei. For (*b*) we get λ from the given half-life and use Eq. 30–3b for the activity. For (*c*) and (*d*) we use Eq. 30–5, and/or make a Table of the times.

SOLUTION (*a*) The atomic mass is 13.0, so 13.0 g will contain 6.02×10^{23} nuclei (Avogadro's number). Since we have only 1.49×10^{-6} g, the number of nuclei N_0 that we have initially is given by the ratio

$$\frac{N_0}{6.02 \times 10^{23}} = \frac{1.49 \times 10^{-6}\ \text{g}}{13.0\ \text{g}},$$

so $N_0 = 6.90 \times 10^{16}$ nuclei.

(*b*) From Eq. 30–6, $\lambda = (0.693)/(600\ \text{s}) = 1.16 \times 10^{-3}\ \text{s}^{-1}$. Then, at $t = 0$ (Eq. 30–3b),

$$\left(\frac{\Delta N}{\Delta t}\right)_0 = \lambda N_0 = \left(1.16 \times 10^{-3}\ \text{s}^{-1}\right)\left(6.90 \times 10^{16}\right) = 8.00 \times 10^{13}\ \text{decays/s}.$$

(*c*)The half-life is 10.0 min, so the decay rate decreases by half every 10.0 min. We can make the Table of activity (in the margin) after given periods of time. After 1.0 h, the activity is 1.25×10^{12} decays/s.

Time (min)	Activity (decays/s)
0	8.00×10^{13}
10	4.00×10^{13}
20	2.00×10^{13}
30	1.00×10^{13}
40	0.500×10^{13}
50	0.250×10^{13}
60	0.125×10^{13}

Easy Alternate Solution **(*c*)** 60 minutes is 6 half-lives, so the activity will decrease to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$ of its original value, or $\left(8.00 \times 10^{13}\right)/(64) = 1.25 \times 10^{12}$ per second.

General Alternate Solution **(*c*)** The general way to find the activity, which works even when the time is not a perfect multiple of $T_{\frac{1}{2}}$, is to use Eq. 30–5. We set $t = 60.0\ \text{min} = 3600\ \text{s}$:

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\lambda t} = \left(8.00 \times 10^{13}\ \text{s}^{-1}\right)e^{-\left(1.16\times10^{-3}\ \text{s}^{-1}\right)(3600\ \text{s})} = 1.23 \times 10^{12}\ \text{s}^{-1}.$$

NOTE The slight discrepancy in results arises because we kept only three significant figures.

(*d*) We want to determine the time t when $\Delta N/\Delta t = 1.00\ \text{s}^{-1}$. From Eq. 30–5, we have

$$e^{-\lambda t} = \frac{(\Delta N/\Delta t)}{(\Delta N/\Delta t)_0} = \frac{1.00\ \text{s}^{-1}}{8.00 \times 10^{13}\ \text{s}^{-1}} = 1.25 \times 10^{-14}.$$

We take the natural log (ln) of both sides (remember $\ln e^{-\lambda t} = -\lambda t$) and divide by λ to find

$$t = -\frac{\ln\left(1.25 \times 10^{-14}\right)}{\lambda} = 2.76 \times 10^4\ \text{s} = 7.67\ \text{h}.$$