* Deriving the Half-Life Formula; Mean Life

We can derive Eq. 30–6 starting from Eq. 30–4 by setting $N = N_0/2$ at $t = T_{\frac{1}{2}}$:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$

so

$$e^{\lambda T_{\frac{1}{2}}} = 2.$$

We take natural logs of both sides ("ln" and "e" are inverse operations, meaning $\ln(e^x) = x$) and find

$$\ln\left(e^{\lambda T_{\frac{1}{2}}}\right) = \ln 2,$$

so

$$\lambda T_{\frac{1}{2}} = \ln 2 = 0.693$$

and

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda},$$

which is Eq. 30–6.

You may find the *mean life* of an isotope quoted. The mean life τ is defined as $\tau = 1/\lambda$, so that Eq. 30–4 can written $N = N_0 e^{-t/\tau}$ just as for RC and LR circuits (Chapters 19 and 21) where τ is called the time constant. Then (see also Eq. 30–6)

$$\tau = \frac{1}{\lambda} = \frac{T_{\frac{1}{2}}}{0.693};$$

the mean life and half-life differ significantly in numerical value, so confusing them can cause serious error (and has).

30–9 Calculations Involving Decay Rates and Half-Life

Let us now consider Examples of what we can determine about a sample of radioactive material if we know the half-life.

EXAMPLE 30–9 **Sample activity.** The isotope $^{14}_{6}C$ has a half-life of 5730 yr. If at some time a sample contains 1.00×10^{22} carbon-14 nuclei, what is the activity of the sample?

APPROACH We first use the half-life to find the decay constant (Eq. 30–6), and use that to find the activity, Eq. 30–3b. The number of seconds in a year is $(60)(60)(24)\left(365\frac{1}{4}\right) = 3.156 \times 10^7\,\text{s}$.

SOLUTION The decay constant λ from Eq. 30–6 is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(5730\,\text{yr})\left(3.156 \times 10^7\,\text{s/yr}\right)} = 3.83 \times 10^{-12}\,\text{s}^{-1}.$$

From Eq. 30–3b, the magnitude of the activity or rate of decay is

$$\frac{\Delta N}{\Delta t} = \lambda N = \left(3.83 \times 10^{-12}\,\text{s}^{-1}\right)\left(1.00 \times 10^{22}\right)$$
$$= 3.83 \times 10^{10}\,\text{decays/s}.$$

Notice that the graph of Fig. 30–10b starts at this value, corresponding to the original value of $N = 1.0 \times 10^{22}$ nuclei in Fig. 30–10a.

NOTE The unit "decays/s" is often written simply as s^{-1} since "decays" is not a unit but refers only to the number. This simple unit of activity is called the becquerel: $1\,\text{Bq} = 1\,\text{decay/s}$, as discussed in Chapter 31.

EXERCISE C Determine the decay constant for radium $\left(T_{\frac{1}{2}} = 1600\,\text{yr}\right)$.