Exponential Decay

Equation 30–3a or b can be solved for N (using calculus) and the result is

$$N = N_0 e^{-\lambda t}, \qquad (30\text{–}4)$$

Radioactive decay law

where N_0 is the number of nuclei present at time $t = 0$, and N is the number remaining after a time t. The symbol e is the natural exponential (encountered earlier in Sections 19–6 and 21–11) whose value is $e = 2.718\cdots$. Thus the number of parent nuclei in a sample decreases exponentially in time, as shown in Fig. 30–10a for $^{14}_{6}C$ decay. Equation 30–4 is called the **radioactive decay law**.

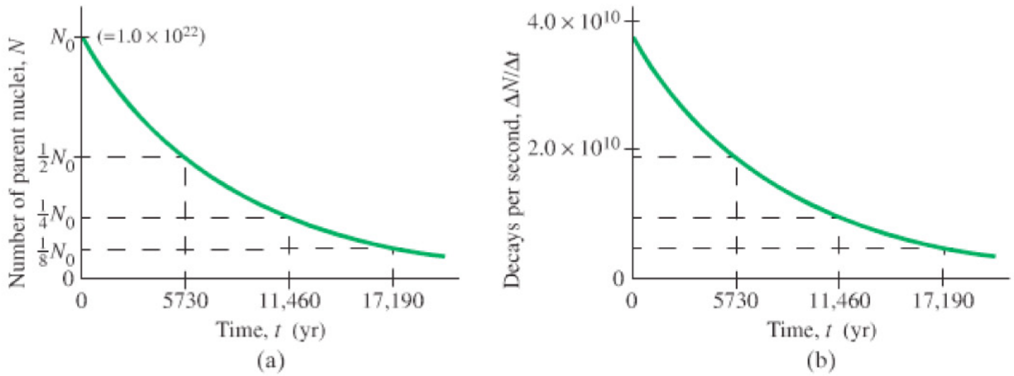

FIGURE 30–10 (a) The number N of parent nuclei in a given sample of $^{14}_{6}C$ decreases exponentially. (b) The number of decays per second also decreases exponentially. The half-life of $^{14}_{6}C$ is 5730 yr, which means that the number of parent nuclei, N, and the rate of decay, $\Delta N/\Delta t$, decreases by half every 5730 yr.

The number of decays per second, $\Delta N/\Delta t$, is called the **activity** (or rate of decay) of the sample. Since $\Delta N/\Delta t$ is proportional to N (see Eq. 30–3b), it, too, decreases exponentially in time at the same rate (Fig. 30–10b). The activity at time t is given by

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\lambda t}, \qquad (30\text{–}5)$$

Activity

where $(\Delta N/\Delta t)_0$ is the activity at $t = 0$.

Half-Life

The rate of decay of any isotope is often specified by giving its "half-life" rather than the decay constant λ. The **half-life** of an isotope is defined as the time it takes for half the original amount of parent isotope in a given sample to decay. For example, the half-life of $^{14}_{6}C$ is about 5730 years. If at some time a piece of petrified wood contains, say, 1.00×10^{22} nuclei of $^{14}_{6}C$, then 5730 years later it will contain only 0.50×10^{22} of these nuclei. After another 5730 years it will contain 0.25×10^{22} nuclei, and so on. This is shown in Fig. 30–10a. Since the rate of decay $\Delta N/\Delta t$ is proportional to N, it, too, decreases by a factor of 2 every half-life (Fig. 30–10b).

The half-lives of known radioactive isotopes vary from as short as 10^{-22} s to about 10^{28} s (about 10^{21} yr). The half-lives of many isotopes are given in Appendix B. It should be clear that the half-life (which we designate $T_{\frac{1}{2}}$) bears an inverse relationship to the decay constant. The longer the half-life of an isotope, the more slowly it decays, and hence λ is smaller. Conversely, very active isotopes (large λ) have very short half-lives. The precise relationship between half-life and decay constant is

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. \qquad (30\text{–}6)$$

Half-life

We derive this in the next (optional) subsection.