shape with a radius that increases with A according to the approximate formula

$$r \approx (1.2 \times 10^{-15}\,\text{m})(A^{\frac{1}{3}}). \qquad \textbf{(30–1)}$$

Nuclear radii

Since the volume of a sphere is $V = \frac{4}{3}\pi r^3$, we see that the volume of a nucleus is approximately proportional to the number of nucleons, $V \propto A$. This is what we would expect if nucleons were like impenetrable billiard balls: if you double the number of balls, you double the total volume. Hence, all nuclei have nearly the same density, and it is enormous (see Example 30–2).

The metric abbreviation for 10^{-15} m is the fermi (after Enrico Fermi) or the femtometer, fm (Table 1–4 or inside the front cover). Thus $1.2 \times 10^{-15}\,\text{m} = 1.2\,\text{fm}$ or 1.2 fermis.

EXAMPLE 30–1 ESTIMATE Nuclear sizes. Estimate the diameter of the following nuclei: (*a*) $^1_1$H, (*b*) $^{40}_{20}$Ca, (*c*) $^{208}_{82}$Pb, (*d*) $^{235}_{92}$U.

APPROACH The radius r of a nucleus is related to its number of nucleons A by Eq. 30–1. The diameter $d = 2r$.

SOLUTION (*a*) For hydrogen, $A = 1$, Eq. 30–1 gives

$$d = \text{diameter} = 2r \approx 2(1.2 \times 10^{-15}\,\text{m})(A^{\frac{1}{3}}) = 2.4 \times 10^{-15}\,\text{m}$$

since $A^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$.

(*b*) For calcium $d = 2r \approx (2.4 \times 10^{-15}\,\text{m})(40)^{\frac{1}{3}} = 8.2 \times 10^{-15}\,\text{m}$.

(*c*) For lead $d \approx (2.4 \times 10^{-15}\,\text{m})(208)^{\frac{1}{3}} = 14 \times 10^{-15}\,\text{m}$.

(*d*) For uranium $d \approx (2.4 \times 10^{-15}\,\text{m})(235)^{\frac{1}{3}} = 15 \times 10^{-15}\,\text{m}$.

The range of nuclear diameters is only from 2.4 fm to 15 fm.

NOTE Because nuclear radii vary as $A^{\frac{1}{3}}$, the largest nuclei have a radius only about 6 times that of the smallest.

EXAMPLE 30–2 ESTIMATE Nuclear and atomic densities. Compare the density of nuclear matter to the density of normal solids.

APPROACH The density of normal liquids and solids is on the order of 10^3 to $10^4\,\text{kg/m}^3$ (see Table 10–1), and because the atoms are close packed, atoms have about this density too. We therefore compare the density (mass per volume) of a nucleus to that of its atom as a whole.

SOLUTION The mass of a proton is greater than the mass of an electron by a factor

$$\frac{1.7 \times 10^{-27}\,\text{kg}}{9.1 \times 10^{-31}\,\text{kg}} \approx 2 \times 10^3.$$

Thus, over 99.9% of the mass of an atom is in the nucleus, and for our estimate we can say the mass of the atom equals the mass of the nucleus, $m_{\text{nucl}}/m_{\text{atom}} = 1$. Atoms have a radius of about 10^{-10} m (Chapter 27) and nuclei on the order of 10^{-15} m (Eq. 30–1). Thus the ratio of nuclear density to atomic density is about

$$\frac{\rho_{\text{nucl}}}{\rho_{\text{atom}}} = \frac{(m_{\text{nucl}}/V_{\text{nucl}})}{(m_{\text{atom}}/V_{\text{atom}})} = \left(\frac{m_{\text{nucl}}}{m_{\text{atom}}}\right)\frac{\frac{4}{3}\pi r^3_{\text{atom}}}{\frac{4}{3}\pi r^3_{\text{nucl}}} \approx (1)\,\frac{(10^{-10})^3}{(10^{-15})^3} = 10^{15}.$$

The nucleus is 10^{15} times more dense than ordinary matter.

The masses of nuclei can be determined from the radius of curvature of fast-moving nuclei (as ions) in a known magnetic field using a mass spectrometer, as discussed in Section 20–11. Indeed the existence of different isotopes of the same element (different number of neutrons) was discovered using this device. Nuclear masses can be specified in **unified atomic mass units** (u). On this scale, a neutral $^{12}_{6}$C atom is given the precise value 12.000000 u. A neutron then has a measured mass of 1.008665 u, a proton 1.007276 u, and a neutral hydrogen atom $^1_1$H (proton plus electron) 1.007825 u. The masses of many nuclides are given in Appendix B. It should be noted that the masses in this Table, as is customary, are for the *neutral atom* (including electrons), and not for a bare nucleus.

CAUTION
Masses are for neutral atom (nucleus plus electrons)