Measurement of the characteristic X-ray spectra has allowed a determination of the inner energy levels of atoms. It has also allowed the determination of Z values for many atoms, since (as we have seen) the wavelength of the shortest X-rays emitted will be inversely proportional to Z^2. Actually, for an electron jumping from, say, the $n = 2$ to the $n = 1$ level, the wavelength is inversely proportional to $(Z - 1)^2$ because the nucleus is shielded by the one electron that still remains in the 1s level. In 1914, H. G. J. Moseley (1887–1915) found that a plot of $\sqrt{1/\lambda}$ versus Z produced a straight line, Fig. 28–12. The Z values of a number of elements were determined by fitting them to such a **Moseley plot**. The work of Moseley put the concept of atomic number on a firm experimental basis.

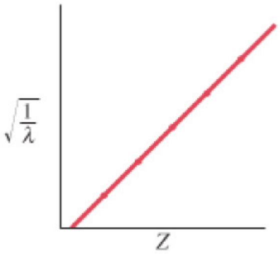

FIGURE 28–12 Plot of $\sqrt{1/\lambda}$ vs. Z for K_α X-ray lines.

EXAMPLE 28–6 **X-ray wavelength.** Estimate the wavelength for an $n = 2$ to $n = 1$ transition in molybdenum $(Z = 42)$. What is the energy of such a photon?

APPROACH We use the Bohr formula, Eq. 27–16 for $1/\lambda$, with Z^2 replaced by $(Z - 1)^2 = (41)^2$.

SOLUTION Equation 27–16 gives

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 e^4 m k^2}{h^3 c}\right)(Z-1)^2\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

where $n = 2$ and $n' = 1$. We substitute in values:

$$\frac{1}{\lambda} = (1.097 \times 10^7\,\mathrm{m}^{-1})(41)^2\left(\frac{1}{1} - \frac{1}{4}\right)$$
$$= 1.38 \times 10^{10}\,\mathrm{m}^{-1}.$$

So $\lambda = 0.072\,\mathrm{nm}$. This is close to the measured value (Fig. 28–11) of 0.071 nm. Each of these photons would have energy (in eV) of:

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\,\mathrm{J\cdot s})(3.00 \times 10^8\,\mathrm{m/s})}{(7.2 \times 10^{-11}\,\mathrm{m})(1.60 \times 10^{-19}\,\mathrm{J/eV})} = 17\,\mathrm{keV}.$$

The denominator includes the conversion factor from joules to eV.

EXAMPLE 28–7 **Determining atomic number.** High-energy photons are used to bombard an unknown material. The strongest peak is found for X-rays emitted with an energy of 66 keV. Guess what the material is.

APPROACH The highest intensity X-rays are generally for the K_α line (see Fig. 28–11) which occurs when photons knock out K shell electrons (the innermost orbit, $n = 1$) and their place is taken by electrons from the L shell $(n = 2)$. We use the Bohr model, and assume the electrons "see" a nuclear charge of $Z - 1$ (screened by one electron) instead of $Z = 1$.

SOLUTION The hydrogen transition $n = 2$ to $n = 1$ would yield about 10.2 eV (see Fig. 27–27 or Example 27–12). Energy E is proportional to Z^2 (Eq. 27–15), or rather $(Z - 1)^2$ because the nucleus is shielded by the one electron in a 1s state (see above), so we can use ratios:

$$\frac{(Z-1)^2}{1^2} = \frac{66 \times 10^3\,\mathrm{eV}}{10.2\,\mathrm{eV}} = 6.5 \times 10^3,$$

so $Z - 1 = \sqrt{6500} = 81$, and $Z = 82$, which makes it lead.