EXERCISE A The area of a rectangle 4.5 cm by 3.25 cm is correctly given by (a) 14.625 cm²; (b) 14.63 cm²; (c) 14.6 cm²; (d) 15 cm².

When adding or subtracting numbers, the final result is no more accurate than the least accurate number used. For example, the result of subtracting 0.57 from 3.6 is 3.0 (and not 3.03).

Keep in mind when you use a calculator that all the digits it produces may not be significant. When you divide 2.0 by 3.0, the proper answer is 0.67, and not some such thing as 0.666666666. Digits should not be quoted in a result, unless they are truly significant figures. However, to obtain the most accurate result, you should normally keep one or more extra significant figures throughout a calculation, and round off only in the final result. (With a calculator, you can keep all its digits in intermediate results.) Note also that calculators sometimes give too few significant figures. For example, when you multiply 2.5×3.2 , a calculator may give the answer as simply 8. But the answer is good to two significant figures, so the proper answer is 8.0. See Fig. 1-6.

EXERCISE B Do 0.00324 and 0.00056 have the same number of significant figures?

Be careful not to confuse significant figures with the number of decimal places.

EXERCISE C For each of the following numbers, state the number of significant figures and the number of decimal places: (a) 1.23; (b) 0.123; (c) 0.0123.

CONCEPTUAL EXAMPLE 1-2 Significant figures. Using a protractor (Fig. 1-7), you measure an angle to be 30°. (a) How many significant figures should you quote in this measurement? (b) Use a calculator to find the cosine of the angle you measured.

RESPONSE (a) If you look at a protractor, you will see that the precision with which you can measure an angle is about one degree (certainly not 0.1°). So you can quote two significant figures, namely, 30° (not 30.0°). (b) If you enter cos 30° in your calculator, you will get a number like 0.866025403. However, the angle you entered is known only to two significant figures, so its cosine is correctly given by 0.87; i.e., you must round your answer to two significant figures.

NOTE We discuss trigonometric functions like cosine in Chapter 3.

Scientific Notation

We commonly write numbers in "powers of ten," or "scientific" notation—for instance 36,900 as 3.69×10^4 , or 0.0021 as 2.1×10^{-3} . One advantage of scientific notation (discussed in Appendix A) is that it allows the number of significant figures to be clearly expressed. For example, it is not clear whether 36,900 has three, four, or five significant figures. With powers of ten notation the ambiguity can be avoided: if the number is known to an accuracy of three significant figures, we write 3.69×10^4 , but if it is known to four, we write 3.690×10^4 .

Percent Error

The significant figures rule is only approximate, and in some cases may underestimate the precision of the answer. Suppose for example we divide 97 by 92:

$$\frac{97}{92} = 1.05 \approx 1.1.$$

Both 97 and 92 have two significant figures, so the rule says to give the answer as 1.1. Yet the numbers 97 and 92 both imply an uncertainty of ±1 if no other uncertainty is stated. Now 92 \pm 1 and 97 \pm 1 both imply an accuracy of about 1% $(1/92 \approx 0.01 = 1\%)$. But the final result to two significant figures is 1.1, with an implied uncertainty of ± 0.1 , which is an uncertainty of $0.1/1.1 \approx 0.1 \approx 10\%$. In this case it is better to give the answer as 1.05 (which is three significant figures). Why? Because 1.05 implies an uncertainty of ± 0.01 which is $0.01/1.05 \approx 0.01 \approx 1\%$, just like the uncertainty in the original numbers 92 and 97.

SUGGESTION: Use the significant figures rule, but consider the % uncertainty too, and add an extra digit if it gives a more realistic estimate of uncertainty.



Calculators err with significant figures

PROBLEM SOLVING

Report only the proper number of significant figures in the final result. Keep extra digits during the calculation.



FIGURE 1-6 These two calculators show the wrong number of significant figures. In (a), 2.0 was divided by 3.0. The correct final result would be 0.67. In (b), 2.5 was multiplied by 3.2. The correct result is 8.0.

FIGURE 1-7 Example 1-2. A protractor used to measure an angle.

