

CONCEPTUAL EXAMPLE 28-3 Possible states for $n = 3$. How many different states are possible for an electron whose principal quantum number is $n = 3$?

RESPONSE For $n = 3$, l can have the values $l = 2, 1, 0$. For $l = 2$, m_l can be $2, 1, 0, -1, -2$, which is five different possibilities. For each of these, m_s can be either up or down ($+\frac{1}{2}$ or $-\frac{1}{2}$); so for $l = 2$, there are $2 \times 5 = 10$ states. For $l = 1$, m_l can be $1, 0, -1$, and since m_s can be $+\frac{1}{2}$ or $-\frac{1}{2}$ for each of these, we have 6 more possible states. Finally, for $l = 0$, m_l can only be 0, and there are only 2 states corresponding to $m_s = +\frac{1}{2}$ and $-\frac{1}{2}$. The total number of states is $10 + 6 + 2 = 18$, as detailed in the following Table:

n	l	m_l	m_s	n	l	m_l	m_s
3	2	2	$+\frac{1}{2}$	3	1	1	$+\frac{1}{2}$
3	2	2	$-\frac{1}{2}$	3	1	1	$-\frac{1}{2}$
3	2	1	$+\frac{1}{2}$	3	1	0	$+\frac{1}{2}$
3	2	1	$-\frac{1}{2}$	3	1	0	$-\frac{1}{2}$
3	2	0	$+\frac{1}{2}$	3	1	-1	$+\frac{1}{2}$
3	2	0	$-\frac{1}{2}$	3	1	-1	$-\frac{1}{2}$
3	2	-1	$+\frac{1}{2}$	3	0	0	$+\frac{1}{2}$
3	2	-1	$-\frac{1}{2}$	3	0	0	$-\frac{1}{2}$
3	2	-2	$+\frac{1}{2}$				
3	2	-2	$-\frac{1}{2}$				

EXERCISE B An electron has $n = 4$, $l = 2$. Which of the following values of m_l are possible: 4, 3, 2, 1, 0, -1, -2, -3, -4?

EXAMPLE 28-4 E and L for $n = 3$. Determine (a) the energy and (b) the orbital angular momentum for an electron in each of the hydrogen atom states of Example 28-3.

APPROACH (a) The energy of a state depends only on n , except for the very small corrections mentioned above, which we will ignore. Energy is calculated as in the Bohr theory, $E_n = -13.6 \text{ eV}/n^2$. For angular momentum we use Eq. 28-3.

SOLUTION Since $n = 3$ for all these states, they all have the same energy,

$$E_3 = -\frac{13.6 \text{ eV}}{(3)^2} = -1.51 \text{ eV}.$$

(b) For $l = 0$, Eq. 28-3 gives

$$L = \sqrt{l(l+1)}\hbar = 0.$$

For $l = 1$,

$$L = \sqrt{1(1+1)}\hbar = \sqrt{2}\hbar = 1.49 \times 10^{-34} \text{ J}\cdot\text{s}.$$

For $l = 2$, $L = \sqrt{2(2+1)}\hbar = \sqrt{6}\hbar$.

NOTE Atomic angular momenta are generally given as a multiple of \hbar ($\sqrt{2}\hbar$ or $\sqrt{6}\hbar$ in this case), rather than in SI units.

EXERCISE C What are the energy and angular momentum of the electron in a hydrogen atom with $n = 6$, $l = 4$?

Although l and m_l do not significantly affect the energy levels in hydrogen, they do affect the electron probability distribution in space. For $n = 1$, l and m_l can only be zero and the electron distribution is as shown in Fig. 28-6.