

EXAMPLE 28-1 **Position uncertainty of electron.** An electron moves in a straight line with a constant speed $v = 1.10 \times 10^6$ m/s which has been measured to a precision of 0.10%. What is the maximum precision with which its position could be simultaneously measured?

APPROACH The momentum is $p = mv$, and the uncertainty in p is $\Delta p = 0.0010p$. The uncertainty principle (Eq. 28-1) gives us the lowest Δx using the equals sign.

SOLUTION The momentum of the electron is

$$p = mv = (9.11 \times 10^{-31} \text{ kg}) \cdot (1.10 \times 10^6 \text{ m/s}) = 1.00 \times 10^{-24} \text{ kg} \cdot \text{m/s}.$$

The uncertainty in the momentum is 0.10% of this, or $\Delta p = 1.0 \times 10^{-27}$ kg·m/s. From the uncertainty principle, the best simultaneous position measurement will have an uncertainty of

$$\Delta x \approx \frac{\hbar}{\Delta p} = \frac{1.06 \times 10^{-34} \text{ J} \cdot \text{s}}{1.0 \times 10^{-27} \text{ kg} \cdot \text{m/s}} = 1.1 \times 10^{-7} \text{ m},$$

or 110 nm.

NOTE This is about 1000 times the diameter of an atom.

EXERCISE A An electron's position is measured with an accuracy of 0.50×10^{-10} m. Find the minimum uncertainty in its momentum and velocity.

EXAMPLE 28-2 **Position uncertainty of a baseball.** What is the uncertainty in position, imposed by the uncertainty principle, on a 150-g baseball thrown at (93 ± 2) mph = (42 ± 1) m/s?

APPROACH The uncertainty in the speed is $\Delta v = 1$ m/s. We multiply Δv by m to get Δp and then use the uncertainty principle, solving for Δx .

SOLUTION The uncertainty in the momentum is

$$\Delta p = m \Delta v = (0.150 \text{ kg})(1 \text{ m/s}) = 0.15 \text{ kg} \cdot \text{m/s}.$$

Hence the uncertainty in a position measurement could be as small as

$$\Delta x = \frac{\hbar}{\Delta p} = \frac{1.06 \times 10^{-34} \text{ J} \cdot \text{s}}{0.15 \text{ kg} \cdot \text{m/s}} = 7 \times 10^{-34} \text{ m}.$$

NOTE This distance is far smaller than any we could imagine observing or measuring. It is trillions of trillions of times smaller than an atom. Indeed, the uncertainty principle sets no relevant limit on measurement for macroscopic objects.

* 28-4 Philosophic Implications; Probability versus Determinism

The classical Newtonian view of the world is a deterministic one (see Section 5-9). One of its basic ideas is that once the position and velocity of an object are known at a particular time, its future position can be predicted if the forces on it are known. For example, if a stone is thrown a number of times with the same initial velocity and angle, and the forces on it remain the same, the path of the projectile will always be the same. If the forces are known (gravity and air resistance, if any), the stone's path can be precisely predicted. This mechanistic view implies that the future unfolding of the universe, assumed to be made up of particulate bodies, is completely determined.