The uncertainty principle does not forbid individual precise measurements, however. For example, in principle we could measure the position of an object exactly. But then its momentum would be completely unknown. Thus, although we might know the position of the object exactly at one instant, we could have no idea at all where it would be a moment later. The uncertainties expressed here are inherent in nature, and reflect the best precision theoretically attainable even with the best instruments.

Another useful form of the uncertainty principle relates energy and time, and we examine this as follows. The object to be detected has an uncertainty in position $\Delta x \approx \lambda$. The photon that detects it travels with speed c, and it takes a time $\Delta t \approx \Delta x/c \approx \lambda/c$ to pass through the distance of uncertainty. Hence, the measured time when our object is at a given position is uncertain by about

$$\Delta t \approx \frac{\lambda}{c}$$
.

Since the photon can transfer some or all of its energy $(= hf = hc/\lambda)$ to our object, the uncertainty in energy of our object as a result is

$$\Delta E \approx \frac{hc}{\lambda}$$

The product of these two uncertainties is

$$(\Delta E)(\Delta t) \approx h.$$

A more careful calculation gives

$$(\Delta E)(\Delta t) \gtrsim \frac{h}{2\pi}.$$
 (28–2)

This form of the uncertainty principle tells us that the energy of an object can be uncertain (or can be interpreted as briefly nonconserved) by an amount ΔE for a time $\Delta t \approx h/(2\pi \Delta E)$.

The quantity $(h/2\pi)$ appears so often in quantum mechanics that for convenience it is given the symbol \(\hat{h} \) ("h-bar"). That is,

$$\hbar = \frac{h}{2\pi} = \frac{6.626 \times 10^{-34} \,\mathrm{J \cdot s}}{2\pi} = 1.055 \times 10^{-34} \,\mathrm{J \cdot s}.$$

By using this notation, Eqs. 28-1 and 28-2 for the uncertainty principle can be written

$$(\Delta x)(\Delta p_x) \gtrsim \hbar$$
 and $(\Delta E)(\Delta t) \gtrsim \hbar$.

We have been discussing the position and velocity of an electron as if it were a particle. But it isn't simply a particle. Indeed, we have the uncertainty principle because an electron-and matter in general-has wave as well as particle properties. What the uncertainty principle really tells us is that if we insist on thinking of the electron as a particle, then there are certain limitations on this simplified view-namely, that the position and velocity cannot both be known precisely at the same time; and even that the electron does not have a precise position and momentum at the same time (because it is not simply a particle). Similarly, the energy can be uncertain in the amount ΔE for a time $\Delta t \approx \hbar/\Delta E$.

Because Planck's constant, h, is so small, the uncertainties expressed in the uncertainty principle are usually negligible on the macroscopic level. But at the level of atomic sizes, the uncertainties are significant. Because we consider ordinary objects to be made up of atoms containing nuclei and electrons, the uncertainty principle is relevant to our understanding of all of nature. The uncertainty principle expresses, perhaps most clearly, the probabilistic nature of quantum mechanics. It thus is often used as a basis for philosophic discussion.

CAUTION Uncertainties not due to instrument deficiency,

but inherent in nature (wave-particle)

UNCERTAINTY PRINCIPLE $(\Delta E \text{ and } \Delta t)$