27–13 de Broglie's Hypothesis Applied to Atoms

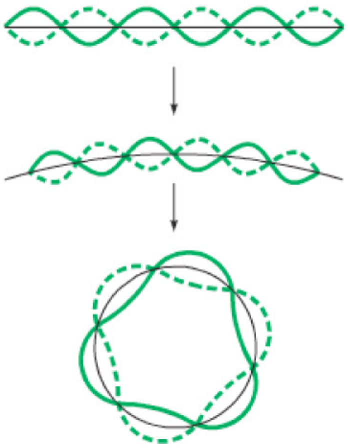


FIGURE 27–28 An ordinary standing wave compared to a circular standing wave.

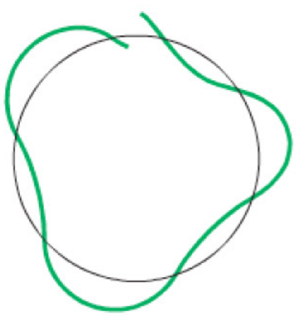
Bohr's theory was largely of an *ad hoc* nature. Assumptions were made so that theory would agree with experiment. But Bohr could give no reason why the orbits were quantized, nor why there should be a stable ground state. Finally, ten years later, a reason was proposed by Louis de Broglie. We saw in Section 27–8 that in 1923, de Broglie proposed that material particles, such as electrons, have a wave nature; and that this hypothesis was confirmed by experiment several years later.

One of de Broglie's original arguments in favor of the wave nature of electrons was that it provided an explanation for Bohr's theory of the hydrogen atom. According to de Broglie, a particle of mass m moving with a nonrelativistic speed v would have a wavelength (Eq. 27–8) of

$$\lambda = \frac{h}{mv}.$$

Each electron orbit in an atom, he proposed, is actually a standing wave. As we saw in Chapter 11, when a violin or guitar string is plucked, a vast number of wavelengths are excited. But only certain ones—those that have nodes at the ends—are sustained. These are the *resonant* modes of the string. Waves with other wavelengths interfere with themselves upon reflection and their amplitudes quickly drop to zero. With electrons moving in circles, according to Bohr's theory, de Broglie argued that the electron wave was a *circular* standing wave that closes on itself, Fig. 27–28. If the wavelength of a wave does not close on itself, as in Fig. 27–29, destructive interference takes place as the wave travels around the loop, and the wave quickly dies out. Thus, the only waves that persist are those for which the circumference of the circular orbit contains a whole number of wavelengths, Fig. 27–30. The circumference of a Bohr orbit of radius r_n is $2\pi r_n$, so we have

FIGURE 27–29 When a wave does not close (and hence interferes destructively with itself), it rapidly dies out.



$$2\pi r_n = n\lambda, \qquad n = 1, 2, 3, \cdots.$$

When we substitute $\lambda = h/mv$, we get $2\pi r_n = nh/mv$, or

$$mvr_n = \frac{nh}{2\pi}.$$

Quantized orbits are a result of wave nature

This is just the *quantum condition* proposed by Bohr on an *ad hoc* basis, Eq. 27–11. It is from this equation that the discrete orbits and energy levels were derived. Thus we have a first explanation for the quantized orbits and energy states in the Bohr model: they are due to the wave nature of the electron, and only

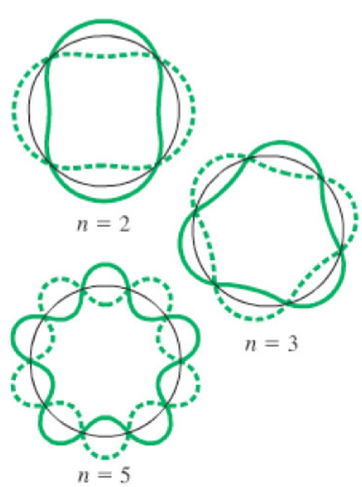


FIGURE 27–30 Standing circular waves for two, three, and five wavelengths on the circumference; n, the number of wavelengths, is also the quantum number.