EXAMPLE 27-14 Absorption wavelength. Use Figure 27-27 to determine the maximum wavelength that hydrogen in its ground state can absorb. What would be the next smaller wavelength that would work?

APPROACH Maximum wavelength corresponds to minimum energy, and this would be the jump from the ground state up to the first excited state (Fig. 27-27). The next smaller wavelength occurs for the jump from the ground state to the second excited state. In each case, the energy difference can be used to find the wavelength.

SOLUTION The energy needed to jump from the ground state to the first excited state is $13.6 \,\mathrm{eV} - 3.4 \,\mathrm{eV} = 10.2 \,\mathrm{eV}$; the required wavelength, as we saw in Example 27-12, is 122 nm. The energy to jump from the ground state to the second excited state is $13.6 \,\mathrm{eV} - 1.5 \,\mathrm{eV} = 12.1 \,\mathrm{eV}$, which corresponds to a wavelength

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_3 - E_1}$$

$$= \frac{(6.63 \times 10^{-34} \,\text{J} \cdot \text{s})(3.00 \times 10^8 \,\text{m/s})}{(12.1 \,\text{eV})(1.60 \times 10^{-19} \,\text{J/eV})} = 103 \,\text{nm}.$$

Additional Examples

EXAMPLE 27–15 Ionization energy. (a) Use the Bohr model to determine the ionization energy of the He+ ion, which has a single electron. (b) Also calculate the maximum wavelength a photon can have to cause ionization.

APPROACH We want to determine the minimum energy required to lift the electron from its ground state and to barely reach the free state at E=0. The ground state energy of He⁺ is given by Eq. 27–15b with n=1and Z = 2.

SOLUTION (a) Since all the symbols in Eq. 27–15b are the same as for the calculation for hydrogen, except that Z is 2 instead of 1, we see that E_1 will be $Z^2 = 2^2 = 4$ times the E_1 for hydrogen. That is,

$$E_1 = 4(-13.6 \,\text{eV}) = -54.4 \,\text{eV}.$$

Thus, to ionize the He⁺ ion should require 54.4 eV, and this value agrees with

(b) The maximum wavelength photon that can cause ionization will have energy $hf = 54.4 \,\mathrm{eV}$ and wavelength

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{(6.63 \times 10^{-34} \,\mathrm{J \cdot s})(3.00 \times 10^8 \,\mathrm{m/s})}{(54.4 \,\mathrm{eV})(1.60 \times 10^{-19} \,\mathrm{J/eV})} = 22.8 \,\mathrm{nm}.$$

If $\lambda > 22.8$ nm, ionization can not occur.

NOTE If the atom absorbed a photon of greater energy (wavelength shorter than 22.8 nm), the atom could still be ionized and the freed electron would have kinetic energy of its own.

In this last Example, we saw that E_1 for the He⁺ ion is four times more negative than that for hydrogen. Indeed, the energy-level diagram for He+ looks just like that for hydrogen, Fig. 27-27, except that the numerical values for each energy level are four times larger. Note, however, that we are talking here about the He+ ion. Normal (neutral) helium has two electrons and its energy level diagram is entirely different.