a definite energy, as the following calculation shows. The total energy equals the sum of the kinetic and potential energies. The potential energy of the electron is given by $\text{PE} = qV = -eV$, where V is the potential due to a point charge $+Ze$ as given by Eq. 17–5: $V = kQ/r = kZe/r$. So

$$\text{PE} = -eV = -k\frac{Ze^2}{r}.$$

The total energy E_n for an electron in the n^{th} orbit of radius r_n is the sum of the kinetic and potential energies:

$$E_n = \frac{1}{2}mv^2 - \frac{kZe^2}{r_n}.$$

When we substitute v from Eq. 27–11 and r_n from Eq. 27–12 into this equation, we obtain

$$E_n = -\frac{2\pi^2 Z^2 e^4 mk^2}{h^2}\frac{1}{n^2} \qquad n = 1, 2, 3, \cdots. \qquad \textbf{(27–15a)}$$

Energy levels

If we evaluate the constant term in Eq. 27–15a and convert it to electron volts, as is customary in atomic physics, we obtain

$$E_n = -(13.6\,\text{eV})\frac{Z^2}{n^2}, \qquad n = 1, 2, 3, \cdots. \qquad \textbf{(27–15b)}$$

The lowest energy level $(n = 1)$ for hydrogen $(Z = 1)$ is

Ground state of hydrogen

$$E_1 = -13.6\,\text{eV}.$$

Since n^2 appears in the denominator of Eq. 27–15b, the energies of the larger orbits in hydrogen $(Z = 1)$ are given by

$$E_n = \frac{-13.6\,\text{eV}}{n^2}.$$

For example,

Excited states (first two)

$$E_2 = \frac{-13.6\,\text{eV}}{4} = -3.40\,\text{eV},$$

$$E_3 = \frac{-13.6\,\text{eV}}{9} = -1.51\,\text{eV}.$$

We see that not only are the orbit radii quantized, but from Eqs. 27–15, so is the energy. The quantum number n that labels the orbit radii also labels the energy levels. The lowest **energy level** or **energy state** has energy E_1, and is called the **ground state**. The higher states, E_2, E_3, and so on, are called **excited states**.

Notice that although the energy for the larger orbits has a smaller numerical value, all the energies are less than zero. Thus, $-3.4\,\text{eV}$ is a greater energy than $-13.6\,\text{eV}$. Hence the orbit closest to the nucleus (r_1) has the lowest total energy. The reason the energies have negative values has to do with the way we defined the zero for potential energy. For two point charges, $\text{PE} = kq_1q_2/r$ corresponds to zero PE when the two charges are infinitely far apart as discussed in Section 17–5. Thus, an electron that can just barely be free from the atom by reaching $r = \infty$ (or, at least, far from the nucleus) with zero KE will have $E = \text{KE} + \text{PE} = 0 + 0 = 0$, corresponding to $n = \infty$ in Eqs. 27–15. If an electron is free and has some kinetic energy, then $E > 0$. To remove an electron that is part of an atom requires an energy input (otherwise atoms would not be stable). Since $E \geq 0$ for a free electron, then it makes sense that an electron bound to an atom must have $E < 0$. That is, energy must be added to bring the electron's total energy up, from a negative value to at least zero in order to free it.

The minimum energy required to remove an electron from the ground state of an atom is called the **binding energy** or **ionization energy**. The ionization energy for hydrogen has been measured to be 13.6 eV, and this corresponds precisely to removing an electron from the lowest state, $E_1 = -13.6\,\text{eV}$, up to $E = 0$ where it can be free.

Binding energy (ionization energy)