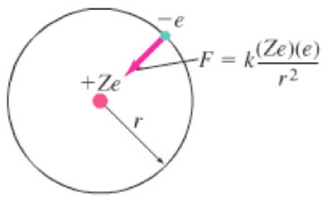


FIGURE 27–25 Electric force (Coulomb's law) keeps the negative electron in orbit around the positively charged nucleus.

An electron in a circular orbit of radius r (Fig. 27–25) would have a centripetal acceleration v^2/r produced by the electrical force of attraction between the negative electron and the positive nucleus. This force is given by Coulomb's law,

$$F = k\,\frac{(Ze)(e)}{r^2},$$

where $k = 1/4\pi\epsilon_0 = 9.00 \times 10^9\,\mathrm{N\cdot m^2/C^2}$. The charge on the electron is $q_1 = -e$, and that on the nucleus is $q_2 = +Ze$, where Z is the number of positive charges† (i.e., protons). For the hydrogen atom, $Z = +1$.

In Newton's second law, $F = ma$, we substitute Coulomb's law for F and $a = v^2/r_n$ for a particular allowed orbit of radius r_n, and obtain

$$F = ma$$

$$k\,\frac{Ze^2}{r_n^2} = \frac{mv^2}{r_n}.$$

We solve this for r_n,

$$r_n = \frac{kZe^2}{mv^2},$$

and then substitute for v from Eq. 27–11 (which says $v = nh/2\pi m r_n$):

$$r_n = \frac{kZe^2 4\pi^2 m r_n^2}{n^2h^2}.$$

We solve for r_n (it appears on both sides, so we cancel one of them) and find

$$r_n = \frac{n^2h^2}{4\pi^2 mkZe^2} = \frac{n^2}{Z}\,r_1 \qquad \textbf{(27–12)}$$

where

$$r_1 = \frac{h^2}{4\pi^2 mke^2}.$$

Equation 27–12 gives the radii of each possible orbit. The smallest orbit is for $n = 1$, and for hydrogen $(Z = 1)$ has the value

$$r_1 = \frac{(1)^2(6.626 \times 10^{-34}\,\mathrm{J\cdot s})^2}{4(3.14)^2(9.11 \times 10^{-31}\,\mathrm{kg})(9.00 \times 10^9\,\mathrm{N\cdot m^2/C^2})(1.602 \times 10^{-19}\,\mathrm{C})^2}$$

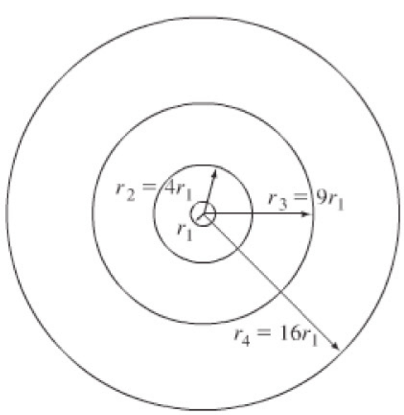
Bohr radius

$$r_1 = 0.529 \times 10^{-10}\,\mathrm{m}. \qquad \textbf{(27–13)}$$

The radius of the smallest orbit in hydrogen, r_1, is sometimes called the **Bohr radius**. From Eq. 27–12, we see that the radii of the larger orbits‡ increase as n^2, so

$$r_2 = 4r_1 = 2.12 \times 10^{-10}\,\mathrm{m},$$
$$r_3 = 9r_1 = 4.76 \times 10^{-10}\,\mathrm{m},$$
$$\vdots$$
$$r_n = n^2 r_1, \qquad n = 1, 2, 3, \cdots.$$

FIGURE 27–26 Possible orbits in the Bohr model of hydrogen; $r_1 = 0.529 \times 10^{-10}\,\mathrm{m}$.



The first four orbits are shown in Fig. 27–26. Notice that, according to Bohr's model, an electron can exist only in the orbits given by Eq. 27–12. There are no allowable orbits in between.

For an atom with $Z \neq 1$, we can write the orbital radii, r_n, using Eq. 27–12:

$$r_n = \frac{n^2}{Z}(0.529 \times 10^{-10}\,\mathrm{m}), \qquad n = 1, 2, 3, \cdots. \qquad \textbf{(27–14)}$$

In each of its possible orbits, the electron in a Bohr model atom would have

†We include Z in our derivation so that we can treat other single-electron ("hydrogenlike") atoms such as the ions He^+ $(Z = 2)$ and Li^{2+} $(Z = 3)$. Helium in the neutral state has two electrons; if one electron is missing, the remaining He^+ ion consists of one electron revolving around a nucleus of charge $+2e$. Similarly, doubly ionized lithium, Li^{2+}, also has a single electron, and in this case $Z = 3$.

‡Be careful not to believe that these well-defined orbits actually exist. Today electrons are better thought of as forming "clouds," as discussed in Chapter 28.