26–10 Relativistic Addition of Velocities

Consider a rocket ship that travels away from the Earth with speed v, and assume that this rocket has fired off a second rocket that travels at speed u' with respect to the first (Fig. 26–10). We might expect that the speed u of rocket 2 with respect to Earth is $u = v + u'$, which in the case shown in Fig. 26–10 is $u = 0.60c + 0.60c = 1.20c$. But, as discussed in Section 26–8, no object can travel faster than the speed of light in any reference frame. Indeed, Einstein showed that since length and time are different in different reference frames, the old addition-of-velocities formula is no longer valid. Instead, the correct formula is

CAUTION

Relative velocities do not add simply, as in classical ($v \ll c$) mechanics

Relativistic addition of velocities formula ($\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ along same line)

$$u = \frac{v + u'}{1 + vu'/c^2} \qquad \left[\begin{array}{l}\vec{\mathbf{u}} \text{ and } \vec{\mathbf{v}} \text{ along} \\ \text{the same direction}\end{array}\right] \qquad \textbf{(26–11)}$$

for motion along a straight line. We derive this formula in Appendix E. If u' is in the opposite direction from v, then u' must have a minus sign in the above equation so $u = (v - u')/(1 - vu'/c^2)$.

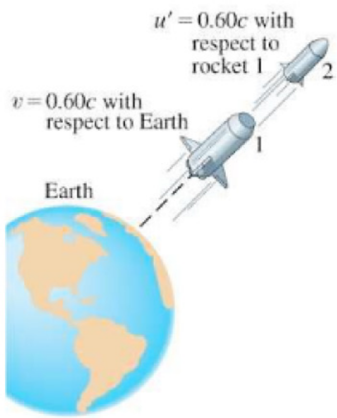

FIGURE 26–10 Rocket 2 is fired from rocket 1 with speed $u' = 0.60c$. What is the speed of rocket 2 with respect to the Earth?

EXAMPLE 26–12 **Relative velocity, relativistically.** Calculate the speed of rocket 2 in Fig. 26–10 with respect to Earth.

APPROACH We add the speed of rocket 2 relative to rocket 1, to the speed of rocket 1 relative to Earth, using the relativistic Eq. 26–11 because the speeds are high and they are along the same line.

SOLUTION Rocket 2 moves with speed $u' = 0.60c$ with respect to rocket 1. Rocket 1 has speed $v = 0.60c$ with respect to Earth. The speed of rocket 2 with respect to Earth is (Eq. 26–11)

$$u = \frac{0.60c + 0.60c}{1 + \dfrac{(0.60c)(0.60c)}{c^2}} = \frac{1.20c}{1.36} = 0.88c.$$

NOTE The speed of rocket 2 relative to Earth is found to be less than c, as it must be.

We can see that Eq. 26–11 reduces to the classical form for velocities small compared to the speed of light since $1 + vu'/c^2 \approx 1$ for v and $u' \ll c$. Thus, $u \approx v + u'$, as in classical physics (Chapter 3).

Let us test our formula in one more case, that of the speed of light. Suppose that rocket 1 in Fig. 26–10 sends out a beam of light so that $u' = c$. Equation 26–11 tells us that the speed of this light with respect to Earth is

$$u = \frac{0.60c + c}{1 + \dfrac{(0.60c)(c)}{c^2}} = \frac{1.60c}{1.60} = c,$$

which is fully consistent with the second postulate of relativity.

EXERCISE F Use Eq. 26–11 to calculate the speed of rocket 2 in Fig. 26–10 relative to Earth if it was shot from rocket 1 at a speed $u' = 3000\ \text{km/s} = 0.010c$. Assume rocket 1 had a speed $v = 6000\ \text{km/s} = 0.020c$.

26–11 The Impact of Special Relativity

A great many experiments have been performed to test the predictions of the special theory of relativity. Within experimental error, no contradictions have been found. Scientists have therefore accepted relativity as an accurate description of nature.