relativistic form for kinetic energy reduces to the classical form, $\text{KE} = \frac{1}{2}mv^2$. This makes relativity a viable theory in that it can predict accurate results at low speed as well as at high. Indeed, the other equations of special relativity also reduce to their classical equivalents at ordinary speeds: length contraction, time dilation, and modifications to momentum as well as kinetic energy, all disappear for $v \ll c$ since $\sqrt{1 - v^2/c^2} \approx 1$.

A useful relation between the total energy E of a particle and its momentum p can also be derived. The momentum of a particle of rest mass m_0 and speed v is given by Eq. 26–4:

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \gamma m_0 v.$$

Relativistic momentum

The total energy is (Eq. 26–7b)

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}.$$

We square this equation (and add a term "$v^2 - v^2$" which is zero, but will help us):

$$E^2 = \frac{m_0^2 c^2 \left(v^2 - v^2 + c^2\right)}{1 - v^2/c^2}$$

$$= p^2c^2 + \frac{m_0^2 c^4 \left(1 - v^2/c^2\right)}{1 - v^2/c^2}$$

or

$$E^2 = p^2c^2 + m_0^2 c^4. \qquad \textbf{(26–10)}$$

Energy related to momentum

Thus, the total energy can be written in terms of the momentum p, or in terms of the kinetic energy (Eq. 26–7a), where we have assumed there is no potential energy.

* When Do We Use Relativistic Formulas?

From a practical point of view, we do not have much opportunity in our daily lives to use the mathematics of relativity. For example, the γ factor, $\gamma = 1/\sqrt{1 - v^2/c^2}$, which appears in many relativistic formulas, has a value of 1.005 when $v = 0.10c$. Thus, for speeds even as high as $0.10c = 3.0 \times 10^7\,\text{m/s}$, the factor $\sqrt{1 - v^2/c^2}$ in relativistic formulas gives a numerical correction of less than 1%. For speeds less than $0.10c$, or unless mass and energy are interchanged, we don't usually need to use the more complicated relativistic formulas, and can use the simpler classical formulas.

If you are given a particle's rest mass m_0 and its kinetic energy KE, you can do a quick calculation to determine if you need to use relativistic formulas or if classical ones are good enough. You simply compute the ratio KE/m_0c^2 because (Eq. 26–6b)

$$\frac{\text{KE}}{m_0 c^2} = \gamma - 1 = \frac{1}{\sqrt{1 - v^2/c^2}} - 1.$$

If this ratio comes out to be less than, say, 0.01, then $\gamma \leq 1.01$ and relativistic equations will correct the classical ones by about 1%. If your expected precision is no better than 1%, classical formulas are good enough. But if your precision is 1 part in 1000 (0.1%) then you would want to use relativistic formulas. If your expected precision is only 10%, you need relativity if $\left(\text{KE}/m_0c^2\right) \gtrsim 0.1$.

EXERCISE E For 1% accuracy, does an electron with $\text{KE} = 100\,\text{eV}$ need to be treated relativistically? [*Hint*: the rest mass of an electron is 0.511 MeV.]