EXAMPLE 26–10 **Mass change in a chemical reaction.** When two moles of hydrogen and one mole of oxygen react to form two moles of water, the energy released is 484 kJ. How much does the mass decrease in this reaction?

APPROACH We use Einstein's great concept of the interchangeability of mass and energy $(E = mc^2)$.

SOLUTION Using Eq. 26–9 we have for the change in mass Δm:

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(-484 \times 10^3\,\mathrm{J})}{(3.00 \times 10^8\,\mathrm{m/s})^2} = -5.38 \times 10^{-12}\,\mathrm{kg}.$$

The initial mass of the system is $0.002\,\mathrm{kg} + 0.016\,\mathrm{kg} = 0.018\,\mathrm{kg}$. Thus the change in mass is relatively very tiny and can normally be neglected. [Conservation of mass is usually a reasonable principle to apply to chemical reactions.]

Units: *eV/c for p* *eV/c^2 for m*

In the tiny world of atoms and nuclei, it is common to quote energies in eV (electron volts) or multiples such as MeV $(10^6\,\mathrm{eV})$. Momentum (see Eq. 26–4) can be quoted in units of eV/c (or MeV/c). And mass can be quoted (from $E = mc^2$) in units of eV/c^2 (or MeV/c^2). Note the use of c to keep the units correct. The rest masses of the electron and the proton are readily shown to be $0.511\,\mathrm{MeV}/c^2$ and $938\,\mathrm{MeV}/c^2$, respectively. See also the Table inside the front cover.

EXAMPLE 26–11 **A 1-TeV proton.** The Tevatron accelerator at Fermilab in Illinois can accelerate protons to a kinetic energy of 1.0 TeV $(10^{12}\,\mathrm{eV})$. What is the speed of such a proton?

APPROACH We solve the kinetic energy formula, Eq. 26–6a, for v.

SOLUTION The rest energy of a proton is $E_0 = 938\,\mathrm{MeV}$ or $9.38 \times 10^8\,\mathrm{eV}$. Compared to the KE of $10^{12}\,\mathrm{eV}$, the rest energy can be neglected, so we simplify Eq. 26–6a to

$$\mathrm{KE} \approx \frac{m_0c^2}{\sqrt{1 - v^2/c^2}}.$$

We solve this for v in the following steps:

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0c^2}{\mathrm{KE}};$$

$$1 - \frac{v^2}{c^2} = \left(\frac{m_0c^2}{\mathrm{KE}}\right)^2;$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_0c^2}{\mathrm{KE}}\right)^2 = 1 - \left(\frac{9.38 \times 10^8\,\mathrm{eV}}{1.0 \times 10^{12}\,\mathrm{eV}}\right)^2;$$

$$v = \sqrt{1 - (9.38 \times 10^{-4})^2}\,c = 0.99999956c.$$

So the proton is traveling at a speed very nearly equal to c.

At low speeds, $v \ll c$, the relativistic formula for KE reduces to the classical one, as we show by using the binomial expansion, $(1 \pm x)^n = 1 \pm nx + n(n-1)x^2/2! + \cdots$. With $n = -\frac{1}{2}$, we expand the square root in Eq. 26–6a

$$\mathrm{KE} = m_0c^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)$$

so that

$$\mathrm{KE} \approx m_0c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots - 1\right)$$
$$\approx \tfrac{1}{2}m_0v^2.$$

The dots in the first expression represent very small terms in the expansion which we neglect since we assumed that $v \ll c$. Thus at low speeds, the