

On a larger scale, the energy produced in nuclear power plants is a result of the loss in rest mass of the uranium fuel as it undergoes the process called fission (Chapter 31). Even the radiant energy we receive from the Sun is an example of $E = mc^2$; the Sun's mass is continually decreasing as it radiates electromagnetic energy outward.

The relation $E = mc^2$ is now believed to apply to all processes, although the changes are often too small to measure. That is, when the energy of a system changes by an amount ΔE , the mass of the system changes by an amount Δm given by

$$\Delta E = (\Delta m)(c^2). \quad (26-9)$$

In a nuclear reaction where an energy E is released (or required to make it go), the masses of the reactants and the products will be different by $\Delta m = \Delta E/c^2$. Even when water is heated on a stove, the mass of the water is assumed to increase very slightly.

EXAMPLE 26-8 Pion's KE. A π^0 meson ($m_0 = 2.40 \times 10^{-28}$ kg) travels at a speed $v = 0.80c = 2.4 \times 10^8$ m/s. What is its kinetic energy? Compare to a classical calculation.

APPROACH Relativistically, kinetic energy is given by Eqs. 26-6. Classically, $\text{KE} = \frac{1}{2}m_0v^2$.

SOLUTION The kinetic energy of our π^0 meson at speed $v = 0.80c$ is (Eq. 26-6a):

$$\begin{aligned} \text{KE} &= m_0c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \\ &= (2.40 \times 10^{-28} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 \left(\frac{1}{(1 - 0.64)^{1/2}} - 1 \right) \\ &= 1.4 \times 10^{-11} \text{ J}. \end{aligned}$$

Notice that the units of m_0c^2 are $\text{kg} \cdot \text{m}^2/\text{s}^2$, which is the joule. A classical calculation would give $\text{KE} = \frac{1}{2}m_0v^2 = \frac{1}{2}(2.4 \times 10^{-28} \text{ kg})(2.4 \times 10^8 \text{ m/s})^2 = 6.9 \times 10^{-12}$ J, about half as much, but this is not a correct result.

NOTE Do not try to calculate the relativistic kinetic energy by using the classical equation with a relativistic mass instead of the rest mass (here $m_{\text{rel}} = m_0/\sqrt{1 - v^2/c^2} = 4.0 \times 10^{-28}$ kg). This would give $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(4.0 \times 10^{-28} \text{ kg})(2.4 \times 10^8 \text{ m/s})^2 = 1.2 \times 10^{-11}$ J, which is incorrect.

PROBLEM SOLVING
Relativistic KE

EXAMPLE 26-9 Energy from nuclear decay. The energy required or released in nuclear reactions and decays comes from a change in mass between the initial and final particles. In one type of radioactive decay (Chapter 30), an atom of uranium ($m = 232.03714$ u) decays to an atom of thorium ($m = 228.02873$ u) plus an atom of helium ($m = 4.00260$ u) where the masses given are in atomic mass units ($1 \text{ u} = 1.6605 \times 10^{-27}$ kg). Calculate the energy released in this decay.

APPROACH The initial mass minus the total final mass gives the mass loss in atomic mass units (u); we convert that to kg, and multiply by c^2 to find the energy released, $\Delta E = \Delta m c^2$.

SOLUTION The initial mass is 232.03714 u, and after the decay the mass is $228.02873 \text{ u} + 4.00260 \text{ u} = 232.03133 \text{ u}$, so there is a decrease in mass of 0.00581 u. This mass, which equals $(0.00581 \text{ u})(1.66 \times 10^{-27} \text{ kg}) = 9.64 \times 10^{-30}$ kg, is changed into energy. By $\Delta E = \Delta m c^2$, we have

$$\Delta E = (9.64 \times 10^{-30} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 = 8.68 \times 10^{-13} \text{ J}.$$

Since $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$, the energy released is 5.4 MeV.

Energy released in nuclear process