26–9 $E = mc^2$; Mass and Energy

If momentum needs to be modified to fit with relativity as we just saw in Eq. 26–4, then we might expect energy too would need to be rethought. Indeed, Einstein not only developed a new formula for kinetic energy, but also found a new relation between mass and energy, and the startling idea that mass is a form of energy.[†]

We can start with the work–energy principle (Chapter 6) and assume it is still valid in relativity. That is, the net work done on a particle is equal to its change in kinetic energy (KE). Using this principle, Einstein showed that at high speeds the formula $\text{KE} = \frac{1}{2}mv^2$ is not correct. Instead, Einstein showed that the kinetic energy of a particle of rest mass m_0 traveling at speed v is given by

Relativistic kinetic energy

$$\text{KE} = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} - m_0c^2. \tag{26–6a}$$

In terms of $\gamma = 1/\sqrt{1 - v^2/c^2}$ we can rewrite Eq. 26–6a as

$$\text{KE} = \gamma m_0c^2 - m_0c^2 = (\gamma - 1)m_0c^2. \tag{26–6b}$$

Equations 26–6 require some interpretation. The first term increases with the speed v of the particle. The second term, m_0c^2, is constant; it is called the **rest energy** E_0 of the particle, and represents a form of energy that a particle has even when at rest. Note that if a particle is at rest ($v = 0$) the first term in Eq. 26–6a becomes m_0c^2, so $\text{KE} = 0$ as it should.

We can rearrange Eq. 26–6b to get

$$\gamma m_0c^2 = m_0c^2 + \text{KE}.$$

We call γm_0c^2 the *total energy* E of the particle (assuming no potential energy), because it equals the rest energy plus the kinetic energy:

Total energy (defined)

$$E = m_0c^2 + \text{KE}. \tag{26–7a}$$

The total energy can also be written, using Eqs. 26–6, as

$$E = \gamma m_0c^2 = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}}. \tag{26–7b}$$

For a particle at rest in a given reference frame, KE is zero in Eqs. 26–7, so the total energy is its rest energy E_0:

MASS RELATED TO ENERGY

$$E_0 = m_0c^2. \tag{26–8}$$

Here we have Einstein's famous formula, usually written simply as $E = mc^2$. This formula mathematically relates the concepts of energy and mass. But if this idea is to have any meaning from a practical point of view, then mass ought to be convertible to other forms of energy and vice versa. Einstein suggested that this might be possible, and indeed changes of mass to other forms of energy, and vice versa, have been experimentally confirmed countless times. The interconversion of mass and energy is most easily detected in nuclear and elementary particle physics. For example, the neutral pion (π^0) of rest mass 2.4×10^{-28} kg is observed to decay into pure electromagnetic radiation (photons). The π^0 completely disappears in the process. The amount of electromagnetic energy produced is found to be exactly equal to that predicted by Einstein's formula, $E = mc^2$. The reverse process is also commonly observed in the laboratory: electromagnetic radiation under certain conditions can be converted into material particles such as electrons (see Section 27–6 on pair production).

Mass and energy interchangeable

[†]The concept of mass being a form of energy emerges nicely from the concept of relativistic mass, Eq. 26–5. When work is done on an object, its kinetic energy increases. The object's speed cannot increase indefinitely because it cannot exceed c; but the object's relativistic mass increases. That is, the work done on an object not only increases its speed but also contributes to increasing its *mass*.