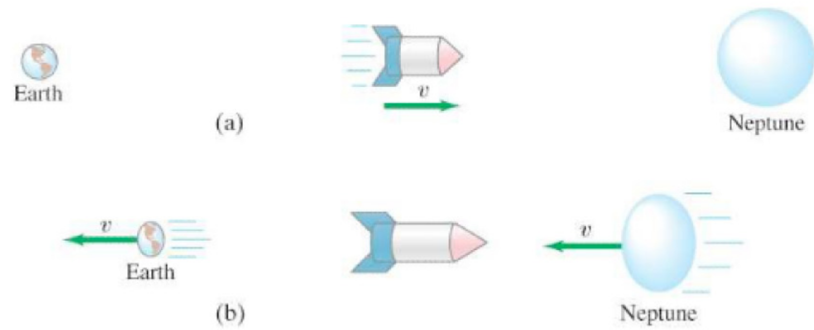

FIGURE 26–7 (a) A spaceship traveling at very high speed from Earth to the planet Neptune, as seen from Earth's frame of reference. (b) According to an observer on the spaceship, Earth and Neptune are moving at the very high speed v: Earth leaves the spaceship, and a time Δt_0 later Neptune arrives at the spaceship.

26–5 Length Contraction

Not only time intervals are different in different reference frames. Space intervals—lengths and distances—are different as well, according to the special theory of relativity, and we illustrate this with a thought experiment.

Observers on Earth watch a spacecraft traveling at speed v from Earth to, say, Neptune, Fig. 26–7a. The distance between the planets, as measured by the Earth observers, is L_0. The time required for the trip, measured from Earth, is

$$\Delta t = \frac{L_0}{v}. \qquad \text{[Earth observer]}$$

In Fig. 26–7b we see the point of view of observers on the spacecraft. In this frame of reference, the spaceship is at rest; Earth and Neptune move† with speed v. The time between departure of Earth and arrival of Neptune (observed from the spacecraft) is the "proper time," since the two events occur at the same point in space (i.e., on the spacecraft). Therefore the time interval is less for the spacecraft observers than for the Earth observers. That is, because of time dilation (Eq. 26–1), the time for the trip as viewed by the spacecraft is

$$\begin{aligned}\Delta t_0 &= \Delta t\sqrt{1 - v^2/c^2} \\ &= \Delta t/\gamma.\end{aligned} \qquad \text{[spacecraft observer]}$$

Because the spacecraft observers measure the same speed but less time between these two events, they also measure the distance as less. If we let L be the distance between the planets as viewed by the spacecraft observers, then $L = v\,\Delta t_0$, which we can rewrite as $L = v\,\Delta t_0 = v\,\Delta t\sqrt{1 - v^2/c^2} = L_0\sqrt{1 - v^2/c^2}$. Thus we have the important result that

Length contraction formula

$$L = L_0\sqrt{1 - v^2/c^2} \tag{26–3a}$$

or, using γ (Eq. 26–2),

$$L = \frac{L_0}{\gamma}. \tag{26–3b}$$

This is a general result of the special theory of relativity and applies to lengths of objects as well as to distance between objects. The result can be stated most simply in words as:

Length contraction: moving objects are shorter (in the direction of motion)

the length of an object is measured to be shorter when it is moving relative to the observer than when it is at rest.

CAUTION
Proper length is measured in reference frame where the two positions are at rest

This is called **length contraction**. The length L_0 in Eq. 26–3 is called the **proper length**. It is the length of the object (or distance between two points whose positions are measured at the same time) as determined by *observers at rest* with respect to it. Equation 26–3 gives the length L that will be measured by observers when the object travels past them at speed v.

†We assume v is much greater than the relative speed of Neptune and Earth, so the latter can be ignored.