(*d*) At a speed half the speed of light

$$\gamma = \frac{1}{\sqrt{1-(0.50)^2}} = 1.15.$$

(*e*) At $v = 0.90c$ we get $\gamma = 2.3$.
(*f*) At $v = 0.990c$ we get $\gamma = 7.1$.

Table 26–1 is a handy summary of these results.

TABLE 26–1 Values of γ

v	γ
0	1.000
$0.01c$	1.000
$0.10c$	1.005
$0.50c$	1.15
$0.90c$	2.3
$0.99c$	7.1

* Global Positioning System (GPS)

PHYSICS APPLIED
Global positioning system (GPS)

Airplanes, cars, boats, and hikers use **global positioning system (GPS)** receivers to tell them quite accurately where they are, at a given moment. The 24 global positioning system satellites send out precise time signals using atomic clocks. Your receiver compares the times received from at least four satellites, all of whose times are carefully synchronized to within 1 part in 10^{13}. By comparing the time differences with the known satellite positions and the fixed speed of light, the receiver can determine how far it is from each satellite and thus where it is on the Earth. It can do this to a typical accuracy of 15 m, if it has been constructed to make corrections such as the one below due to special relativity.

CONCEPTUAL EXAMPLE 26–4 **A relativity correction to GPS.** GPS satellites move at about $4\,\text{km/s} = 4000\,\text{m/s}$. Show that a good GPS receiver needs to correct for time dilation if it is to produce results consistent with atomic clocks accurate to 1 part in 10^{13}.

RESPONSE Let us calculate the magnitude of the time dilation effect by inserting $v = 4000\,\text{m/s}$ into Eq. 26–1a:

$$\Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,\Delta t_0$$

$$= \frac{1}{\sqrt{1-\left(\frac{4\times10^3\,\text{m/s}}{3\times10^8\,\text{m/s}}\right)^2}}\,\Delta t_0$$

$$= \frac{1}{\sqrt{1-1.8\times10^{-10}}}\,\Delta t_0.$$

We use the binomial expansion: $(1\pm x)^n \approx 1 \pm nx$ for $x \ll 1$ (see Appendix A) which here is $(1-x)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}x$. That is

$$\Delta t = \left(1+\tfrac{1}{2}(1.8\times10^{-10})\right)\Delta t_0 = \left(1+9\times10^{-11}\right)\Delta t_0.$$

The time "error" divided by the time interval is

$$\frac{(\Delta t - \Delta t_0)}{\Delta t_0} = 1 + 9\times10^{-11} - 1 = 9\times10^{-11} \approx 1\times10^{-10}.$$

Time dilation, if not accounted for, would introduce an error of about 1 part in 10^{10}, which is 1000 times greater than the precision of the atomic clocks. Not correcting for time dilation means a receiver could give much poorer position accuracy.

NOTE GPS devices must make other corrections as well, including effects associated with General Relativity.