Twin Paradox

Twin paradox

Not long after Einstein proposed the special theory of relativity, an apparent paradox was pointed out. According to this **twin paradox**, suppose one of a pair of 20-year-old twins takes off in a spaceship traveling at very high speed to a distant star and back again, while the other twin remains on Earth. According to the Earth twin, the astronaut twin will age less. Whereas 20 years might pass for the Earth twin, perhaps only 1 year (depending on the spacecraft's speed) would pass for the traveler. Thus, when the traveler returns, the earthbound twin could expect to be 40 years old whereas the traveling twin would be only 21.

This is the viewpoint of the twin on the Earth. But what about the traveling twin? If all inertial reference frames are equally good, won't the traveling twin make all the claims the Earth twin does, only in reverse? Can't the astronaut twin claim that since the Earth is moving away at high speed, time passes more slowly on Earth and the twin on Earth will age less? This is the opposite of what the Earth twin predicts. They cannot both be right, for after all the spacecraft returns to Earth and a direct comparison of ages and clocks can be made.

There is, however, no contradiction here. The consequences of the special theory of relativity—in this case, time dilation—can be applied only by observers in an inertial reference frame. The Earth is such a frame (or nearly so), whereas the spacecraft is not. The spacecraft accelerates at the start and end of its trip and when it turns around at the far point of its journey. During the acceleration, the twin on the spacecraft is not in an inertial frame. In between, the astronaut twin may be in an inertial frame (and is justified in saying the Earth twin's clocks run slow), but it is not always the same frame. So she cannot predict their relative ages when she returns to Earth. The Earth twin stays in the same inertial frame, and we can thus trust her predictions based on special relativity. Thus, there is no paradox. The prediction of the Earth twin that the traveling twin ages less is the proper one.

* Additional Example—Using γ

EXAMPLE 26–3 **γ for various speeds.** Determine the value of γ for a speed v equal to (*a*) 0, (*b*) $0.010c$, (*c*) $0.10c$, (*d*) $0.50c$, (*e*) $0.90c$, (*f*) $0.990c$.

APPROACH We simply plug into Eq. 26–2.

SOLUTION (*a*) For $v = 0, \gamma = 1/1 = 1$ exactly.
(*b*) For $v = 0.010c = 3.0 \times 10^6\,\mathrm{m/s}$ (a pretty high speed):

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{0.010c}{c}\right)^2}}$$

$$= \frac{1}{\sqrt{1 - (0.010)^2}} = \frac{1}{\sqrt{0.99990}} = 1.00005.$$

Unless v is given to more significant figures, $\gamma = 1.0$ here. We see that γ is never less than 1.0 and will only exceed 1.0 significantly at higher speeds.
(*c*) For a speed 10 times higher, $v = 0.10c$, we get

$$\gamma = \frac{1}{\sqrt{1 - (0.10)^2}} = \frac{1}{\sqrt{0.99}} = 1.005.$$