EXAMPLE 26–1 **Lifetime of a moving muon.** (*a*) What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c = 1.80 \times 10^8\,\mathrm{m/s}$ with respect to the laboratory? Its mean lifetime at rest is $2.20\,\mu\mathrm{s} = 2.20 \times 10^{-6}\,\mathrm{s}$. (*b*) How far does a muon travel in the laboratory, on average, before decaying?

APPROACH If an observer were to move along with the muon (the muon would be at rest to this observer), the muon would have a mean life of 2.2×10^{-6} s. To an observer in the lab, the muon lives longer because of time dilation. We find the mean lifetime using Eq. 26–1a and the average distance from $d = v\,\Delta t$.

SOLUTION (*a*) From Eq. 26–1a with $v = 0.60c$, we have

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.20 \times 10^{-6}\,\mathrm{s}}{\sqrt{1 - \frac{0.36c^2}{c^2}}} = \frac{2.20 \times 10^{-6}\,\mathrm{s}}{\sqrt{0.64}} = 2.8 \times 10^{-6}\,\mathrm{s}.$$

(*b*) Relativity predicts that a muon would travel an average distance $d = v\,\Delta t = (0.60)(3.0 \times 10^8\,\mathrm{m/s})(2.8 \times 10^{-6}\,\mathrm{s}) = 500\,\mathrm{m}$, and this is the distance that is measured experimentally in the laboratory.

NOTE At a speed of $1.8 \times 10^8\,\mathrm{m/s}$, classical physics would tell us that with a mean life of $2.2\,\mu\mathrm{s}$, an average muon would travel $d = vt = (1.8 \times 10^8\,\mathrm{m/s})(2.2 \times 10^{-6}\,\mathrm{s}) = 400\,\mathrm{m}$. This is shorter than the distance measured.

EXERCISE B What is the muon's mean life in Example 26–1 if v is (*a*) $0.10c$, (*b*) $0.90c$?

The proper time Δt_0 is in a reference frame where the two events occur at the same point in space

We need to clarify how to use Eq. 26–1, and the meaning of Δt and Δt_0. The equation is true only when Δt_0 represents the time interval between the two events in a reference frame where the two events occur at *the same point in space* (as in Fig. 26–6a where the two events are the light flash being sent and being received). This time interval, Δt_0, is called the **proper time**. Then Δt in Eq. 26–1 represents the time interval between the two events as measured in a reference frame moving with speed v with respect to the first. In Example 26–1 above, Δt_0 (and not Δt) was set equal to 2.2×10^{-6} s because it is only in the rest frame of the muon that the two events ("birth" and "decay") occur at the same point in space. The proper time Δt_0 is the shortest time between the events any observer can measure. In any other moving reference frame, the time Δt is greater.

EXAMPLE 26–2 **Time dilation at 100 km/h.** Let's check time dilation for everyday speeds. A car traveling 100 km/h covers a certain distance in 10.00 s according to the driver's watch. What does an observer at rest on Earth measure for the time interval?

APPROACH The car's speed relative to Earth is $100\,\mathrm{km/h} = (1.00 \times 10^5\,\mathrm{m})/(3600\,\mathrm{s}) = 27.8\,\mathrm{m/s}$. The driver is at rest in the reference frame of the car, so we set $\Delta t_0 = 10.00\,\mathrm{s}$ in the time dilation formula.

SOLUTION We use Eq. 26–1a:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10.00\,\mathrm{s}}{\sqrt{1 - \left(\frac{27.8\,\mathrm{m/s}}{3.00 \times 10^8\,\mathrm{m/s}}\right)^2}} = \frac{10.00\,\mathrm{s}}{\sqrt{1 - (8.59 \times 10^{-15})}}.$$

If you put these numbers into a calculator, you will obtain $\Delta t = 10.00\,\mathrm{s}$, since the denominator differs from 1 by such a tiny amount. Indeed, the time measured by an observer on Earth would show no difference from that measured by the driver, even with the best instruments. A computer that could calculate to a large number of decimal places would show Δt is greater than Δt_0 by about 4×10^{-14} s.