where $L = v\,\Delta t/2$, and therefore

$$c = \frac{2\sqrt{D^2 + L^2}}{\Delta t} = \frac{2\sqrt{D^2 + v^2(\Delta t)^2/4}}{\Delta t}.$$

We square both sides,

$$c^2 = \frac{4D^2}{(\Delta t)^2} + v^2,$$

and solve for Δt, to find

$$(\Delta t)^2 = \frac{4D^2}{c^2 - v^2}$$

$$\Delta t = \frac{2D}{c\sqrt{1 - v^2/c^2}}.$$

We combine this equation with the formula on page 734, $\Delta t_0 = 2D/c$:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}. \tag{26–1a}$$

Time dilation formula

Since $\sqrt{1 - v^2/c^2}$ is always less than 1, we see that $\Delta t > \Delta t_0$. That is, the time interval between the two events (the sending of the light, and its reception on the spaceship) is *greater* for the observer on Earth than for the observer on the spaceship. This is a general result of the theory of relativity, and is known as **time dilation**. Stated simply, the time dilation effect says that

> **clocks moving relative to an observer are measured by that observer to run more slowly (as compared to clocks at rest).**

Time dilation: moving clocks run slow

However, we should not think that the clocks are somehow at fault. Time is actually measured to pass more slowly in any moving reference frame as compared to your own. This remarkable result is an inevitable outcome of the two postulates of the theory of relativity.

The factor $1/\sqrt{1 - v^2/c^2}$ occurs so often in relativity that we often give it the shorthand symbol γ, and write Eq. 26–1a as

$$\Delta t = \gamma\,\Delta t_0 \tag{26–1b}$$

Time dilation

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{26–2}$$

γ defined

Note that γ is never less than one. At normal speeds, $\gamma = 1$ to a few decimal places; in general, $\gamma \geq 1$.

The concept of time dilation may be hard to accept, for it contradicts our experience. We can see from Eq. 26–1 that the time dilation effect is indeed negligible unless v is reasonably close to c. If v is much less than c, then the term v^2/c^2 is much smaller than the 1 in the denominator of Eq. 26–1a, and then $\Delta t \approx \Delta t_0$ (see Example 26–2). The speeds we experience in everyday life are much smaller than c, so it is little wonder we don't ordinarily notice time dilation. Experiments have tested the time dilation effect, and have confirmed Einstein's predictions. In 1971, for example, extremely precise atomic clocks were flown around the world in jet planes. The speed of the planes $(10^3\,\mathrm{km/h})$ was much less than c, so the clocks had to be accurate to nanoseconds $(10^{-9}\,\mathrm{s})$ in order to detect any time dilation. They were this accurate, and they confirmed Eq. 26–1 to within experimental error. Time dilation had been confirmed decades earlier, however, by observation on "elementary particles" which have very small masses (typically 10^{-30} to $10^{-27}\,\mathrm{kg}$) and so require little energy to be accelerated to speeds close to the speed of light, c. Many of these elementary particles are not stable and decay after a time into lighter particles. One example is the muon, whose mean lifetime is $2.2\,\mu\mathrm{s}$ when at rest. Careful experiments showed that when a muon is traveling at high speeds, its lifetime is measured to be longer than when it is at rest, just as predicted by the time dilation formula.

Why we don't usually notice time dilation

Experimental confirmation