NOTE In both cases, we determined the limit set by diffraction. The resolution for a visible-light Earth-bound telescope is not this good because of aberrations and, more importantly, turbulence in the atmosphere. In fact, large-diameter objectives are not justified by increased resolution, but by their greater light-gathering ability—they allow more light in, so fainter objects can be seen. Radiotelescopes are not hindered by atmospheric turbulence, and the resolution found in (b) is a good estimate.

Diffraction sets an ultimate limit on the detail that can be seen on any object. In Eq. 25-8 for resolving power, the focal length of a lens cannot practically be made less than (approximately) the radius of the lens, and even that is very difficult (see the lensmaker's equation, Eq. 23-10). In this best case, Eq. 25-8 gives, with  $f \approx D/2$ .

$$RP \approx \frac{\lambda}{2}.$$
 (25–9)

Thus we can say, to within a factor of 2 or so, that

it is not possible to resolve detail of objects smaller than the wavelength of the radiation being used.

This is an important and useful rule of thumb.

Compound lenses in microscopes are now designed so well that the actual limit on resolution is often set by diffraction—that is, by the wavelength of the light used. To obtain greater detail, one must use radiation of shorter wavelength. The use of UV radiation can increase the resolution by a factor of perhaps 2. Far more important, however, was the discovery in the early twentieth century that electrons have wave properties (Chapter 27) and that their wavelengths can be very small. The wave nature of electrons is utilized in the electron microscope (Section 27-9), which can magnify 100 to 1000 times more than a visible-light microscope because of the much shorter wavelengths. X-rays, too, have very short wavelengths and are often used to study objects in great detail (Section 25-11).

## 25-9 Resolution of the Human Eye and **Useful Magnification**

The resolution of the human eye is limited by several factors, all of roughly the same order of magnitude. The resolution is best at the fovea, where the cone spacing is smallest, about  $3 \mu m$  (= 3000 nm). The diameter of the pupil varies from about 0.1 cm to about 0.8 cm. So for  $\lambda = 550$  nm (where the eye's sensitivity is greatest), the diffraction limit is about  $\theta \approx 1.22 \lambda/D \approx 8 \times 10^{-5}$  rad to  $6 \times 10^{-4}$  rad. The eye is about 2 cm long, giving a resolving power (Eq. 25–8) of  $s \approx (2 \times 10^{-2} \,\mathrm{m})(8 \times 10^{-5} \,\mathrm{rad}) \approx 2 \,\mu\mathrm{m}$  at best, to about 15  $\mu\mathrm{m}$  at worst (pupil small). Spherical and chromatic aberration also limit the resolution to about 10 μm. The net result is that the eye can resolve objects whose angular separation is about

$$5 \times 10^{-4} \, \text{rad}$$

at best. This corresponds to objects separated by 1 cm at a distance of about 20 m.

The typical near point of a human eye is about 25 cm. At this distance, the eye can just resolve objects that are  $(25 \text{ cm})(5 \times 10^{-4} \text{ rad}) \approx 10^{-4} \text{ m} = \frac{1}{10} \text{ mm}$ apart. Since the best light microscopes can resolve objects no smaller than about 200 nm at best (Eq. 25-9 for violet light,  $\lambda = 400$  nm), the useful magnification [= (resolution by naked eye)/(resolution by microscope)] is limited to about

$$\frac{10^{-4}\,\text{m}}{200\times 10^{-9}\,\text{m}}\approx 500\times.$$

In practice, magnifications of about 1000× are often used to minimize eyestrain. Any greater magnification would simply make visible the diffraction pattern produced by the microscope objective.

Resolution limited to \(\lambda\)

Best eve resolution

Maximum useful microscope magnification