25–8 Resolution of Telescopes and Microscopes; the λ Limit

You might think that a microscope or telescope could be designed to produce any desired magnification, depending on the choice of focal lengths and quality of the lenses. But this is not possible, because of diffraction. An increase in magnification above a certain point merely results in magnification of the diffraction patterns. This can be highly misleading since we might think we are seeing details of an object when we are really seeing details of the diffraction pattern. To examine this problem, we apply the Rayleigh criterion: two objects (or two nearby points on one object) are just resolvable if they are separated by an angle θ (Fig. 25–30) given by Eq. 25–7:

$$\theta = \frac{1.22\lambda}{D}.$$

This formula is valid for either a microscope or a telescope, where D is the diameter of the objective lens. For a telescope, the resolution is specified by stating θ as given by this equation.†

For a microscope, it is more convenient to specify the actual distance, s, between two points that are just barely resolvable: see Fig. 25–30. Since objects are normally placed near the focal point of the microscope objective, the angle subtended by two objects is $\theta = s/f$, or $s = f\theta$. If we combine this with Eq. 25–7, we obtain for the **resolving power** (**RP**) of a microscope

Resolving power

$$\text{RP} = s = f\theta = \frac{1.22\lambda f}{D}, \qquad \textbf{(25–8)}$$

where f is the objective lens' focal length (not frequency). This distance s is called the resolving power of the lens because it is the minimum separation of two object points that can just be resolved, assuming the highest quality lens since this limit is imposed by the wave nature of light. A smaller RP means better resolution, better detail.

FIGURE 25–32 The 300-meter radiotelescope in Arecibo, Puerto Rico, uses radio waves (Fig. 22–8) instead of visible light.

EXAMPLE 25–12 Telescope resolution (radio wave vs. visible light). What is the theoretical minimum angular separation of two stars that can just be resolved by (*a*) the 200-inch telescope on Palomar Mountain (Fig. 25–21c); and (*b*) the Arecibo radiotelescope (Fig. 25–32), whose diameter is 300 m and whose radius of curvature is also 300 m. Assume $\lambda = 550\,\text{nm}$ for the visible-light telescope in part (*a*), and $\lambda = 4\,\text{cm}$ (the shortest wavelength at which the radiotelescope has been operated) in part (*b*).

APPROACH We apply the Rayleigh criterion (Eq. 25–7) for each telescope.

SOLUTION (*a*) Since $D = 200\,\text{in.} = 5.1\,\text{m}$, we have from Eq. 25–7 that

$$\theta = \frac{1.22\lambda}{D} = \frac{(1.22)(5.50\times10^{-7}\,\text{m})}{(5.1\,\text{m})} = 1.3\times10^{-7}\,\text{rad},$$

or 0.75×10^{-5} deg. (Note that this is equivalent to resolving two points less than 1 cm apart from a distance of 100 km!)

(*b*) For radio waves with $\lambda = 0.04\,\text{m}$, the resolution is

$$\theta = \frac{(1.22)(0.04\,\text{m})}{(300\,\text{m})} = 1.6\times10^{-4}\,\text{rad}.$$

The resolution is less because the wavelength is so much larger, but the larger objective is a plus.

† Earth-bound telescopes with large-diameter objectives are usually limited not by diffraction but by other effects such as turbulence in the atmosphere. The resolution of a high-quality microscope, on the other hand, normally *is* limited by diffraction; microscope objectives are complex compound lenses containing many elements of small diameter (since f is small), thus reducing aberrations.