

FIGURE 24-26 Spectra produced by a grating: (a) two wavelengths, 400 nm and 700 nm; (b) white light. The second order will normally be dimmer than the first order. (Higher orders are not shown.) If grating spacing is small enough, the second and higher orders will be missing.

will be a distinct spectrum of colors spread out over a certain angular width, Fig. 24-26b. Because a diffraction grating spreads out light into its component wavelengths, the resulting pattern is called a spectrum.

**EXAMPLE 24-6** Diffraction grating: lines. Determine the angular positions of the first- and second-order maxima for light of wavelength 400 nm and 700 nm incident on a grating containing 10,000 lines/cm.

**APPROACH** First we find the distance d between grating lines: if the grating has N lines in 1 m, then the distance between lines must be d = 1/N meters. Then we use Eq. 24–4 to find the angles for the two wavelengths for m = 1 and 2. **SOLUTION** The grating contains  $1.00 \times 10^4$  lines/cm =  $1.00 \times 10^6$  lines/m, which means the distance between lines is  $d = (1/1.00 \times 10^6) \,\mathrm{m} =$  $1.00 \times 10^{-6}$  m =  $1.00 \,\mu$ m. In first order (m = 1), the angles are

$$\sin \theta_{400} = \frac{m\lambda}{d} = \frac{(1)(4.00 \times 10^{-7} \,\mathrm{m})}{1.00 \times 10^{-6} \,\mathrm{m}} = 0.400$$
$$\sin \theta_{700} = \frac{(1)(7.00 \times 10^{-7} \,\mathrm{m})}{1.00 \times 10^{-6} \,\mathrm{m}} = 0.700$$

so  $\theta_{400} = 23.6^{\circ}$  and  $\theta_{700} = 44.4^{\circ}$ . In second order,

$$\sin \theta_{400} = \frac{2\lambda}{d} = \frac{(2)(4.00 \times 10^{-7} \,\mathrm{m})}{1.00 \times 10^{-6} \,\mathrm{m}} = 0.800$$

$$\sin \theta_{700} = \frac{(2)(7.00 \times 10^{-7} \,\mathrm{m})}{1.00 \times 10^{-6} \,\mathrm{m}} = 1.40$$

so  $\theta_{400} = 53.1^{\circ}$ . But the second order does not exist for  $\lambda = 700 \, \text{nm}$  because  $\sin \theta$  cannot exceed 1. No higher orders will appear.

**EXAMPLE 24-7** Spectra overlap. White light containing wavelengths from 400 nm to 750 nm strikes a grating containing 4000 lines/cm. Show that the blue at  $\lambda = 450 \,\mathrm{nm}$  of the third-order spectrum overlaps the red at 700 nm of the second order.

**APPROACH** We use Eq. 24–4 to calculate the angular positions of the m=3blue maximum and the m = 2 red one.

**SOLUTION** The grating spacing is  $d = (1/4000) \text{ cm} = 2.50 \times 10^{-6} \text{ m}$ . The blue of the third order occurs at an angle  $\theta$  given by

$$\sin \theta = \frac{m\lambda}{d} = \frac{(3)(4.50 \times 10^{-7} \,\mathrm{m})}{(2.50 \times 10^{-6} \,\mathrm{m})} = 0.540.$$

Red in second order occurs at

$$\sin \theta = \frac{(2)(7.00 \times 10^{-7} \,\mathrm{m})}{(2.50 \times 10^{-6} \,\mathrm{m})} = 0.560,$$

which is a greater angle; so the second order overlaps into the beginning of the third-order spectrum.