



FIGURE 23-38 Deriving the lens equation for a diverging lens.

We can derive the lens equation for a diverging lens using Fig. 23-38. Triangles IAI' and OAO' are similar; and triangles IFI' and AFB are similar. Thus (noting that length $AB = h_o$)

$$\frac{h_i}{h_o} = \frac{d_i}{d_o} \quad \text{and} \quad \frac{h_i}{h_o} = \frac{f - d_i}{f}.$$

When we equate the right sides of these two equations and simplify, we obtain

$$\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f}.$$

This equation becomes the same as Eq. 23-8 if we make f and d_i negative. That is, we take f to be *negative for a diverging lens*, and d_i negative when the image is on the same side of the lens as the light comes from. Thus Eq. 23-8 will be valid for both converging and diverging lenses, and for *all* situations, if we use the following **sign conventions**:

1. The focal length is positive for converging lenses and negative for diverging lenses.
2. The object distance is positive if the object is on the side of the lens from which the light is coming (this is usually the case, although when lenses are used in combination, it might not be so); otherwise, it is negative.
3. The image distance is positive if the image is on the opposite side of the lens from where the light is coming; if it is on the same side, d_i is negative. Equivalently, the image distance is positive for a real image and negative for a virtual image.
4. The height of the image, h_i , is positive if the image is upright, and negative if the image is inverted relative to the object. (h_o is always taken as positive.)

The **magnification**, m , of a lens is defined as the ratio of the image height to object height, $m = h_i/h_o$. From Figs. 23-37 and 23-38 and the conventions just stated (for which we'll need a minus sign below), we have

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \quad (23-9)$$

CAUTION
Focal length is negative for diverging lens

PROBLEM SOLVING
SIGN CONVENTIONS for lenses

Magnification of a lens

For an upright image the magnification is positive, and for an inverted image the magnification is negative.

From sign convention 1, it follows that the power (Eq. 23-7) of a converging lens, in diopters, is positive, whereas the power of a diverging lens is negative. A converging lens is sometimes referred to as a **positive lens**, and a diverging lens as a **negative lens**.

PROBLEM SOLVING Thin Lenses

1. Draw a **ray diagram**, as precise as possible, but even a rough one can serve as confirmation of analytic results. Choose one point on the object and draw at least two, preferably three, of the easy-to-draw rays described in Figs. 23-34 and 23-36. The image point is where the rays intersect.
2. For analytic solutions, solve for unknowns in the

thin lens equation (Eq. 23-8) and the **magnification equation** (Eq. 23-9). The thin lens equation involves reciprocals—don't forget to take the reciprocal.

3. Follow the **sign conventions** above.
4. Check that your analytic answers are **consistent** with your ray diagram.