(see vector diagram, Fig. 3–28)

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}. \quad \textbf{(3–6)}$$

Follow the subscripts

By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3–6 are the same, whereas the outer subscripts on the right of Eq. 3–6 (the B and the S) are the same as the two subscripts for the sum vector on the left, $\mathbf{v}_{BS}$. By following this convention (first subscript for the object, second for the reference frame), one can write down the correct equation relating velocities in different reference frames.† Equation 3–6 is valid in general and can be extended to three or more velocities. For example, if a fisherman on the boat walks with a velocity $\vec{\mathbf{v}}_{FB}$ relative to the boat, his velocity relative to the shore is $\vec{\mathbf{v}}_{FS} = \vec{\mathbf{v}}_{FB} + \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$. The equations involving relative velocity will be correct when adjacent inner subscripts are identical and when the outermost ones correspond exactly to the two on the velocity on the left of the equation. But this works only with plus signs (on the right), not minus signs.

It is often useful to remember that for any two objects or reference frames, A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\vec{\mathbf{v}}_{BA} = -\vec{\mathbf{v}}_{AB}. \quad \textbf{(3–7)}$$

For example, if a train is traveling 100 km/h relative to the Earth in a certain direction, objects on the Earth (such as trees) appear to an observer on the train to be traveling 100 km/h in the opposite direction.

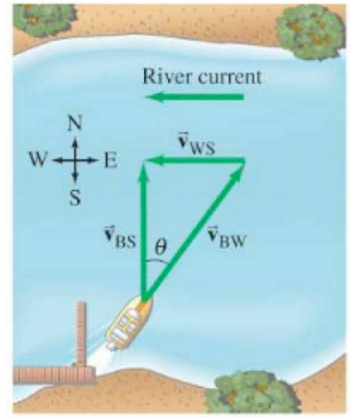


FIGURE 3–28 To move directly across the river, the boat must head upstream at an angle θ. Velocity vectors are shown as green arrows:

$\vec{\mathbf{v}}_{BS}$ = velocity of **B**oat with respect to the **S**hore,
$\vec{\mathbf{v}}_{BW}$ = velocity of **B**oat with respect to the **W**ater,
$\vec{\mathbf{v}}_{WS}$ = velocity of **W**ater with respect to the **S**hore (river current).

CONCEPTUAL EXAMPLE 3–10 **Crossing a river.** A man in a small motor boat is trying to cross a river that flows due west with a strong current. The man starts on the south bank and is trying to reach the north bank directly north from his starting point. Should he (*a*) head due north, (*b*) head due west, (*c*) head in a northwesterly direction, (*d*) head in a northeasterly direction?

RESPONSE If the man heads straight across the river, the current will drag the boat downstream (westward). To overcome the river's westward current, the boat must acquire an eastward component of velocity as well as a northward component. Thus the boat must (*d*) head in a northeasterly direction (see Fig. 3–28). The actual angle depends on the strength of the current and how fast the boat moves relative to the water. If the current is weak and the motor is strong, then the boat can head almost, but not quite, due north.

EXAMPLE 3–11 **Heading upstream.** A boat's speed in still water is $v_{BW} = 1.85\,\mathrm{m/s}$. If the boat is to travel directly across a river whose current has speed $v_{WS} = 1.20\,\mathrm{m/s}$, at what upstream angle must the boat head? (See Fig. 3–29.)

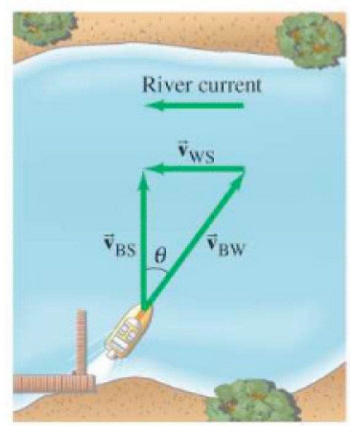
APPROACH We reason as in Example 3–10, and use subscripts as in Eq. 3–6. Figure 3–29 has been drawn with $\vec{\mathbf{v}}_{BS}$, the velocity of the **B**oat relative to the **S**hore, pointing directly across the river since this is how the boat is supposed to move. (Note that $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$.) To accomplish this, the boat needs to head upstream to offset the current pulling it downstream.

SOLUTION Vector $\vec{\mathbf{v}}_{BW}$ points upstream at an angle θ as shown. From the diagram,

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\,\mathrm{m/s}}{1.85\,\mathrm{m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a $40.4°$ angle.

FIGURE 3–29 Example 3–11.



† We thus would know by inspection that (for example) the equation $\vec{\mathbf{v}}_{BW} = \vec{\mathbf{v}}_{BS} + \vec{\mathbf{v}}_{WS}$ is wrong: the inner subscripts are not the same, and the outer ones on the right are not the same as the subscripts on the left.