Mirror Equation and Magnification

Image points can be determined, roughly, by drawing the three rays as just described, Fig. 23–13; but it is difficult to draw small angles for the "paraxial" rays as we assumed. For more accurate results, we now derive an equation that gives the image distance if the object distance and radius of curvature of the mirror are known. To do this, we refer to Fig. 23–14. The **object distance**, d_o, is the distance of the object (point O) from the center of the mirror. The **image distance**, d_i, is the distance of the image (point I) from the center of the mirror. The height of the object OO′ is called h_o and the height of the image, I′I, is h_i. Two rays leaving O′ are

Distances d_o and d_i are measured from center of mirror

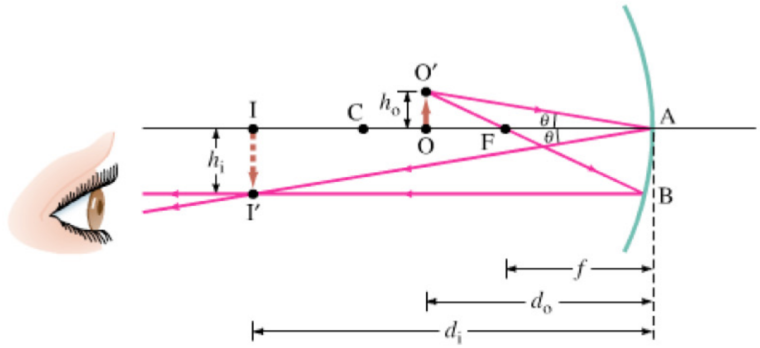

FIGURE 23–14 Diagram for deriving the mirror equation. For the derivation, we assume the mirror size is small compared to its radius of curvature.

shown: O′FBI′ (same as ray 2 in Fig. 23–13) and O′AI′, which is a fourth type of ray that reflects at the center of the mirror and can also be used to find an image point. The ray O′AI′ obeys the law of reflection, so the two right triangles O′AO and I′AI are similar. Therefore, we have

$$\frac{h_o}{h_i} = \frac{d_o}{d_i}.$$

For the other ray shown, O′FBI′, the triangles O′FO and AFB are also similar because the angles are equal and we use the approximation $AB = h_i$ (mirror small compared to its radius). Furthermore $FA = f$, the focal length of the mirror, so

$$\frac{h_o}{h_i} = \frac{OF}{FA} = \frac{d_o - f}{f}.$$

The left sides of the two preceding expressions are the same, so we can equate the right sides:

$$\frac{d_o}{d_i} = \frac{d_o - f}{f}.$$

We now divide both sides by d_o and rearrange to obtain

Mirror equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad \textbf{(23–2)}$$

This is the equation we were seeking. It is called the **mirror equation** and relates the object and image distances to the focal length f (where $f = r/2$).

The **magnification**, m, of a mirror is defined as the height of the image divided by the height of the object. From our first set of similar triangles above, or the first equation on this page, we can write:

Magnification of curved mirror

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \textbf{(23–3)}$$

The minus sign in Eq. 23–3 is inserted as a convention. Indeed, we must