FIGURE 3–27 Examples of projectile motion—sparks (small hot glowing pieces of metal), water, and fireworks. All exhibit the parabolic path characteristic of projectile motion, although the effects of air resistance can be seen to alter the path of some trajectories.

* 3–7 Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a parabola, if we ignore air resistance and assume that $\vec{\mathbf{g}}$ is constant. To show this, we need to find y as a function of x by eliminating t between the two equations for horizontal and vertical motion (Eq. 2–11b), and we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$
$$y = v_{y0}t - \tfrac{1}{2}gt^2.$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2.$$

If we write $v_{x0} = v_0 \cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$, we can also write

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2.$$

In either case, we see that y as a function of x has the form

$$y = Ax - Bx^2,$$

where A and B are constants for any specific projectile motion. This is the well-known equation for a parabola. See Figs. 3–17 and 3–27.

The idea that projectile motion is parabolic was, in Galileo's day, at the forefront of physics research. Today we discuss it in Chapter 3 of introductory physics!

* 3–8 Relative Velocity

We now consider how observations made in different reference frames are related to each other. For example, consider two trains approaching one another, each with a constant speed of 80 km/h with respect to the Earth. Observers on the Earth beside the tracks will measure 80 km/h for the speed of each train. Observers on either of the trains (a different reference frame) will measure a speed of 160 km/h for the other train approaching them.

Similarly, when one car traveling 90 km/h passes a second car traveling in the same direction at 75 km/h, the first car has a speed relative to the second car of 90 km/h − 75 km/h = 15 km/h.

When the velocities are along the same line, simple addition or subtraction is sufficient to obtain the relative velocity. But if they are not along the same line, we must use vector addition. We emphasize, as mentioned in Section 2–1, that when specifying a velocity, it is important to specify what the reference frame is.

➥ **PROBLEM SOLVING**

Subscripts for adding velocities: first subscript for the object; second subscript for the reference frame

When determining relative velocity, it is easy to make a mistake by adding or subtracting the wrong velocities. It is important, therefore, to draw a diagram and use a careful labeling process. Each velocity is labeled by *two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity*. For example, suppose a boat is to cross a river to the opposite side, as shown in Fig. 3–28. We let $\vec{\mathbf{v}}_{BW}$ be the velocity of the **B**oat with respect to the **W**ater. (This is also what the boat's velocity would be relative to the shore if the water were still.) Similarly, $\vec{\mathbf{v}}_{BS}$ is the velocity of the **B**oat with respect to the **S**hore, and $\vec{\mathbf{v}}_{WS}$ is the velocity of the **W**ater with respect to the **S**hore (this is the river current). Note that $\vec{\mathbf{v}}_{BW}$ is what the boat's motor produces (against the water), whereas $\vec{\mathbf{v}}_{BS}$ is equal to $\vec{\mathbf{v}}_{BW}$ plus the effect of the current, $\vec{\mathbf{v}}_{WS}$. Therefore, the velocity of the boat relative to the shore is