$2\theta_0 = 180° - 60.6° = 119.4°$ is also a solution (see Appendix A–7). In general we will have two solutions (see Fig. 3–25b), which in the present case are given by

$$\theta_0 = 30.3° \quad \text{or} \quad 59.7°.$$

Either angle gives the same range. Only when $\sin 2\theta_0 = 1$ (so $\theta_0 = 45°$) is there a single solution (that is, both solutions are the same).

Additional Example: Slightly more Complicated, but Fun

EXAMPLE 3–9 **A punt.** Suppose the football in Example 3–5 was a punt and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0$, $y_0 = 0$.

PHYSICS APPLIED

Sports

APPROACH The x and y motions are again treated separately. But we cannot use the range formula from Example 3–8 because it is valid only if y (final) $= y_0$, which is not the case here. Now we have $y_0 = 0$, and the football hits the ground where $y = -1.00\,\text{m}$ (see Fig. 3–26). We choose our time interval to start when the ball leaves his foot $(t = 0, y_0 = 0, x_0 = 0)$ and end just before the ball hits the ground $(y = -1.00\,\text{m})$. We can get x from Eq. 2–11b, $x = v_{x0}t$, since we know that $v_{x0} = 16.0\,\text{m/s}$ from Example 3–5. But first we must find t, the time at which the ball hits the ground, which we obtain from the y motion.

PROBLEM SOLVING

Do not use any formula unless you are sure its range of validity fits the problem. The range formula does not apply here because $y \neq y_0$

SOLUTION With $y = -1.00\,\text{m}$ and $v_{y0} = 12.0\,\text{m/s}$ (see Example 3–5), we use the equation

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2,$$

and obtain

$$-1.00\,\text{m} = 0 + (12.0\,\text{m/s})t - (4.90\,\text{m/s}^2)t^2.$$

We rearrange this equation into standard form so we can use the quadratic formula (Appendix A–4; also Example 2–15):

$$(4.90\,\text{m/s}^2)t^2 - (12.0\,\text{m/s})t - (1.00\,\text{m}) = 0.$$

Using the quadratic formula gives

$$t = \frac{12.0\,\text{m/s} \pm \sqrt{(12.0\,\text{m/s})^2 - 4(4.90\,\text{m/s}^2)(-1.00\,\text{m})}}{2(4.90\,\text{m/s}^2)}$$
$$= 2.53\,\text{s} \quad \text{or} \quad -0.081\,\text{s}.$$

The second solution would correspond to a time prior to the kick, so it doesn't apply. With $t = 2.53\,\text{s}$ for the time at which the ball touches the ground, the horizontal distance the ball traveled is (using $v_{x0} = 16.0\,\text{m/s}$ from Example 3–5):

$$x = v_{x0}t = (16.0\,\text{m/s})(2.53\,\text{s}) = 40.5\,\text{m}.$$

Our assumption in Example 3–5 that the ball leaves the foot at ground level results in an underestimate of about 1.3 m in the distance traveled.

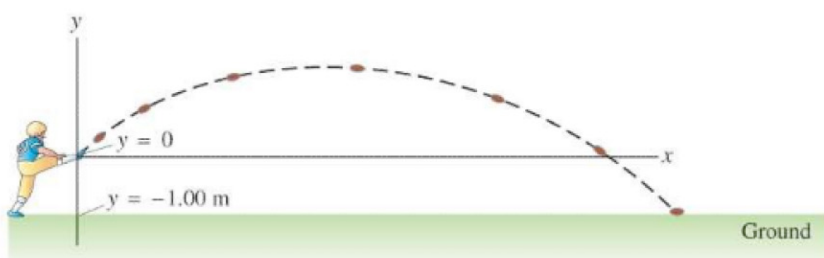

FIGURE 3–26 Example 3–9: the football leaves the punter's foot at $y = 0$, and reaches the ground where $y = -1.00\,\text{m}$.