a right triangle), that

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2}$$
$$= I_0\sqrt{R^2 + (X_L - X_C)^2}.$$

Thus, from Eq. 21–14, the total impedance Z is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}. \quad \textbf{(21–15)}$$

Total impedance

Also from Fig. 21–41, we can find the phase angle ϕ between voltage and current:

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0(X_L - X_C)}{I_0 R} = \frac{X_L - X_C}{R} \quad \textbf{(21–16a)}$$

and

$$\cos\phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}. \quad \textbf{(21–16b)}$$

Figure 21–41 was drawn for the case $X_L > X_C$, and the current lags the source voltage by ϕ. When the reverse is true, $X_L < X_C$, then ϕ in Eqs. 21–16 is less than zero, and the current leads the source voltage.

We saw earlier that power is dissipated only by a resistance; none is dissipated by inductance or capacitance. Therefore, the average power $\overline{P} = I_{\text{rms}}^2 R$. But from Eq. 21–16b, $R = Z\cos\phi$. Therefore

$$\overline{P} = I_{\text{rms}}^2 Z\cos\phi = I_{\text{rms}} V_{\text{rms}} \cos\phi. \quad \textbf{(21–17)}$$

Power factor

The factor $\cos\phi$ is referred to as the *power factor* of the circuit.

EXAMPLE 21–18 ***LRC* circuit.** Suppose $R = 25.0\,\Omega$, $L = 30.0\,\text{mH}$, and $C = 12.0\,\mu\text{F}$ in Fig. 21–39, and they are connected to a 90.0-V ac (rms) 500-Hz source. Calculate (*a*) the current in the circuit, and (*b*) the voltmeter readings (rms) across each element.

APPROACH To obtain the current, we need to determine the impedance (Eq. 21–15 plus Eqs. 21–11b and 21–12b), and then use $I_{\text{rms}} = V_{\text{rms}}/Z$. Voltage drops across each element are found using Ohm's law or equivalent for each element: $V_R = IR$, $V_L = IX_L$, and $V_C = IX_C$.

SOLUTION (*a*) First, we find the reactance of the inductor and capacitor at $f = 500\,\text{Hz} = 500\,\text{s}^{-1}$:

$$X_L = 2\pi f L = 94.2\,\Omega, \qquad X_C = \frac{1}{2\pi f C} = 26.5\,\Omega.$$

Then the total impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(25.0\,\Omega)^2 + (94.2\,\Omega - 26.5\,\Omega)^2} = 72.2\,\Omega.$$

From the impedance version of Ohm's law, Eq. 21–14,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{90.0\,\text{V}}{72.2\,\Omega} = 1.25\,\text{A}.$$

(*b*) The rms voltage across each element is

$$(V_R)_{\text{rms}} = I_{\text{rms}} R = (1.25\,\text{A})(25.0\,\Omega) = 31.2\,\text{V}$$
$$(V_L)_{\text{rms}} = I_{\text{rms}} X_L = (1.25\,\text{A})(94.2\,\Omega) = 118\,\text{V}$$
$$(V_C)_{\text{rms}} = I_{\text{rms}} X_C = (1.25\,\text{A})(26.5\,\Omega) = 33.1\,\text{V}.$$

NOTE These voltages do *not* add up to the source voltage, 90.0 V (rms). Indeed, the rms voltage across the inductance *exceeds* the source voltage. This can happen because the different voltages are out of phase with each other, and at any instant one voltage can be negative, to compensate for a large positive voltage of another. The rms voltages, however, are always positive by definition. Although the rms voltages need not add up to the source voltage, the instantaneous voltages at any time must add up to the source voltage at that instant.