* 21–13 *LRC* Series AC Circuit

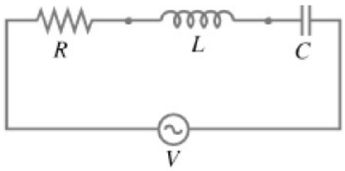

FIGURE 21–39 An *LRC* circuit.

Let us examine a circuit containing all three elements in series: a resistor R, an inductor L, and a capacitor C, Fig. 21–39. If a given circuit contains only two of these elements, we can still use the results of this Section by setting $R = 0$, $X_L = 0$, or $X_C = 0$, as needed. We let V_R, V_L, and V_C represent the voltage across each element at a *given instant* in time; and V_{R0}, V_{L0}, and V_{C0} represent the *maximum* (peak) values of these voltages. The voltage across each of the elements will follow the phase relations we discussed in the previous Section. At any instant the voltage V supplied by the source will be, by Kirchhoff's loop rule,

$$V = V_R + V_L + V_C. \tag{21–13}$$

CAUTION
Peak voltages do not add to yield source voltage

Because the various voltages are not in phase, they do not reach their peak values at the same time, so the peak voltage of the source V_0 will *not* equal $V_{R0} + V_{L0} + V_{C0}$.

* Phasor Diagrams

Let us now examine an *LRC* circuit in detail. The current at any instant must be the same at all points in the circuit. Thus the currents in each element are in phase with each other, even though the voltages are not. We choose our origin in time ($t = 0$) so that the current I at any time t is

$$I = I_0 \cos 2\pi f t.$$

We analyze an *LRC* circuit using a **phasor diagram**. Arrows (treated like vectors) are drawn in an xy coordinate system to represent each voltage. The length of each arrow represents the magnitude of the peak voltage across each element:

$$V_{R0} = I_0 R, \qquad V_{L0} = I_0 X_L, \qquad \text{and} \qquad V_{C0} = I_0 X_C.$$

V_{R0} is in phase with the current and is initially ($t = 0$) drawn along the positive x axis, as is the current. V_{L0} leads the current by 90°, so it leads V_{R0} by 90° and is initially drawn along the positive y axis. V_{C0} lags the current by 90°, so V_{C0} is drawn initially along the negative y axis. See Fig. 21–40a. If we let the vector diagram rotate counterclockwise at frequency f, we get the diagram shown in Fig. 21–40b; after a time, t, each arrow has rotated through an angle $2\pi f t$. Then the projections of each arrow on the x axis represent the voltages across each element at the instant t (Fig. 21–40c). For example $I = I_0 \cos 2\pi f t$.

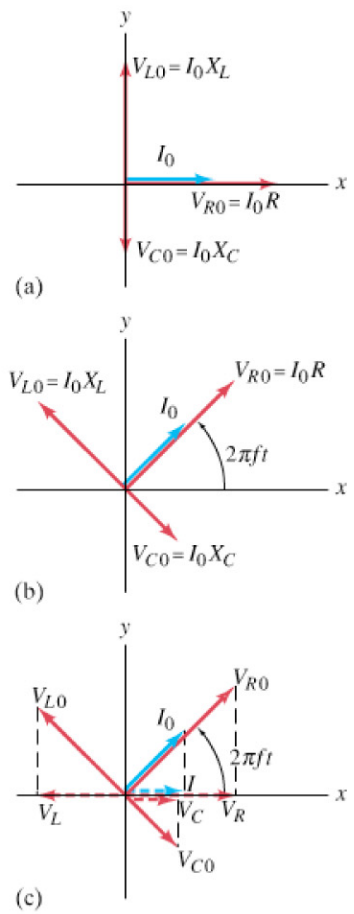

FIGURE 21–40 Phasor diagram for a series *LRC* circuit.

The sum of the projections of the three voltage vectors represents the instantaneous voltage across the whole circuit, V. Therefore, the vector sum of these vectors will be the vector that represents the peak source voltage, V_0, as shown in Fig. 21–41 where it is seen that V_0 makes an angle ϕ with I_0 and V_{R0}. As time passes, V_0 rotates with the other vectors, so the instantaneous voltage V (projection of V_0 on the x axis) is (see Fig. 21–41)

$$V = V_0 \cos(2\pi f t + \phi).$$

The voltage V across the whole circuit must, of course, equal the source voltage (Fig. 21–39). Thus the voltage from the source is out of phase with the current by an angle ϕ.

Impedance

From this analysis we can now determine the total **impedance** Z of the circuit, which is defined by the relation

$$V_{\text{rms}} = I_{\text{rms}} Z, \qquad \text{or} \qquad V_0 = I_0 Z. \tag{21–14}$$

From Fig. 21–41 we see, using the Pythagorean theorem (V_0 is the hypotenuse of

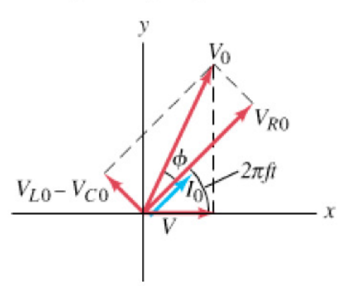

FIGURE 21–41 Phasor diagram for a series *LRC* circuit showing the sum vector, V_0.