EXAMPLE 21–15 **Solenoid time constant.** A solenoid has an inductance of 87.5 mH and a resistance of 0.250 Ω. Find (*a*) the time constant for this circuit, and (*b*) how long it would take for the current to go from zero to 63% of its final (maximum) value when connected to a battery of voltage V.

APPROACH The time constant is $\tau = L/R$. Then we use the equation for I, setting $I = 0.63 I_{max}$ and solving it for t.

SOLUTION (*a*) By definition, $\tau = L/R = (87.5 \times 10^{-3}\,\mathrm{H})/(0.250\,\Omega) = 0.350\,\mathrm{s}$. (*b*) We saw above that $I = (V/R)(1 - e^{-t/\tau})$. We wish to find t such that $I = (0.63)(V/R)$ where $V/R = I_{max}$. This occurs when $t = \tau = L/R$, so $t = 0.350\,\mathrm{s}$.

* 21–12 AC Circuits and Reactance

We have previously discussed circuits that contain combinations of resistor, capacitor, and inductor, but only when they are connected to a dc source of emf or to no source (as in the discharge of a capacitor in an RC circuit). Now we discuss these circuit elements when they are connected to a source of alternating voltage that produces an alternating current (ac).

First we examine, one at a time, how a resistor, a capacitor, and an inductor behave when connected to a source of alternating voltage, represented by the symbol

[alternating voltage]

which produces a sinusoidal voltage of frequency f. We assume in each case that the emf gives rise to a current

$$I = I_0 \cos 2\pi f t,$$

where t is time and I_0 is the peak current. Remember (Section 18–7) that $V_{rms} = V_0/\sqrt{2}$ and $I_{rms} = I_0/\sqrt{2}$ (Eq. 18–8).

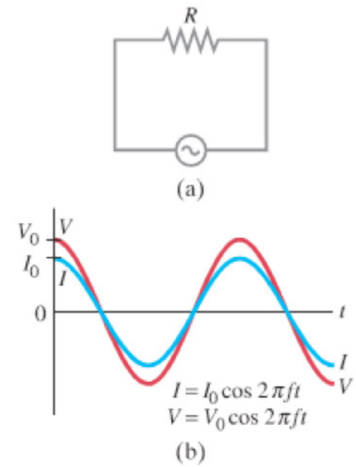


FIGURE 21–35 (a) Resistor connected to an ac source. (b) Current (blue curve) is in phase with the voltage (red) across a resistor.

* Resistor

When an ac source is connected to a resistor as in Fig. 21–35a, the current increases and decreases with the alternating emf according to Ohm's law, $I = V/R$. Figure 21–35b shows the voltage (red curve) and the current (blue curve). Because the current is zero when the voltage is zero and the current reaches a peak when the voltage does, we say that the current and voltage are **in phase**. Energy is transformed into heat (Section 18–7), at an average rate $\overline{P} = \overline{IV} = I_{rms}^2 R = V_{rms}^2/R$.

Resistor: current and voltage are in phase

* Inductor

In Fig. 21–36a an inductor of inductance L (symbol —ᴍᴍᴍ—) is connected to the ac source. We ignore any resistance it might have (it is usually small). The voltage applied to the inductor will be equal to the "back" emf generated in the inductor by the changing current as given by Eq. 21–9. This is because the sum of the emfs around any closed circuit must be zero, as Kirchhoff's rule tells us. Thus

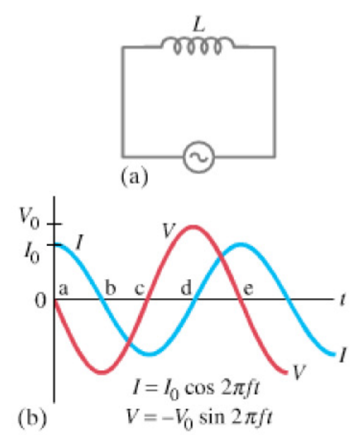
$$V - L\frac{\Delta I}{\Delta t} = 0 \qquad \text{or} \qquad V = L\frac{\Delta I}{\Delta t},$$

where V is the sinusoidally varying voltage of the source and $L\,\Delta I/\Delta t$ is the voltage induced in the inductor. According to this equation, I is increasing most rapidly when V has its maximum value, $V = V_0$. And I will be decreasing most rapidly when $V = -V_0$. These two instants correspond to points d and b on the graph of voltage versus time in Fig. 21–36b. By going point by point in this manner, the curve of I versus t as compared to that for V versus t can be constructed, and they are shown by the blue and red lines, respectively, in Fig. 21–36b. Notice that the current reaches its peaks (and troughs) $\frac{1}{4}$ cycle after the voltage does. We say that

the current lags the voltage by 90° in an inductor.

Because the current and voltage in an inductor are out of phase by 90°, the product IV (= power) is as often positive as it is negative (Fig. 21–36b). So no energy is transformed in an inductor on the average; and no energy is dissipated as thermal energy.

FIGURE 21–36 (a) Inductor connected to an ac source. (b) Current (blue curve) lags voltage (red curve) by a quarter cycle or 90°.



Inductor: current lags voltage