* 21–10 Energy Stored in a Magnetic Field

In Section 17–9 we saw that the energy stored in a capacitor is equal to $\frac{1}{2}CV^2$. By using a similar argument, it can be shown that the energy U stored in an inductance L, carrying a current I, is

$$U = \text{energy} = \tfrac{1}{2}LI^2.$$

Just as the energy stored in a capacitor can be considered to reside in the electric field between its plates, so the energy in an inductor can be considered to be stored in its magnetic field.

To write the energy in terms of the magnetic field, we use the result of Example 21–14 that the inductance of a solenoid is $L = \mu_0 N^2 A/l$. Now the magnetic field B in a solenoid is related to the current I (see Eq. 20–8) by $B = \mu_0 NI/l$. Thus, $I = Bl/\mu_0 N$, and

$$U = \text{energy} = \tfrac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al.$$

We can think of this energy as residing in the volume enclosed by the windings, which is Al. Then the energy per unit volume, or **energy density**, is

Energy density in magnetic field

$$u = \text{energy density} = \frac{1}{2}\frac{B^2}{\mu_0}. \qquad \textbf{(21–10)}$$

This formula, which was derived for the special case of a solenoid, can be shown to be valid for any region of space where a magnetic field exists. If a ferromagnetic material is present, μ_0 is replaced by μ. This equation is analogous to that for an electric field, $\frac{1}{2}\epsilon_0 E^2$, Section 17–9.

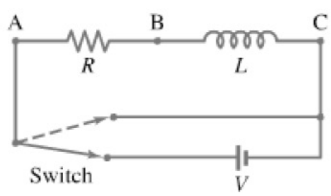


FIGURE 21–33 *LR* circuit.

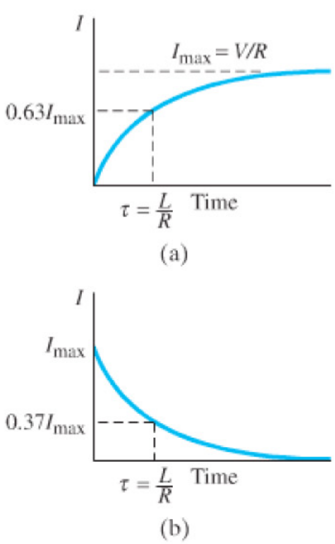
* 21–11 *LR* Circuit

Any inductor will have some resistance. We represent this situation by drawing the inductance L and the resistance R separately, as in Fig. 21–33. The resistance R could also include a separate resistor connected in series. Now we ask, what happens when a dc source is connected in series to such an LR circuit? At the instant the switch connecting the battery is closed, the current starts to flow. It is opposed by the induced emf in the inductor because of the changing current. However, as soon as current starts to flow, there is a voltage drop across the resistance ($V = IR$). Hence, the voltage drop across the inductance is reduced, and there is then less impedance to the current flow from the inductance. The current thus rises gradually, as shown in Fig. 21–34a, and approaches the steady value $I_{max} = V/R$ when all the voltage drop is across the resistance. The shape of the curve for I as a function of time is

$$I = \left(\frac{V}{R}\right)\left(1 - e^{-t/\tau}\right), \qquad [LR \text{ circuit with emf}]$$

where e is the number $e = 2.718\cdots$ (see Section 19–6) and $\tau = L/R$ is the **time constant** of the circuit. When $t = \tau$, then $\left(1 - e^{-1}\right) = 0.63$, so τ is the time required for the current to reach $0.63I_{max}$.

FIGURE 21–34 (a) Growth of current in an *LR* circuit when connected to a battery; (b) decay of current when the *LR* circuit is shorted out (battery is out of the circuit).



If the battery is suddenly removed from the circuit (dashed line in Fig. 21–33), the current decreases as shown in Fig. 21–34b. This is an exponential decay curve given by

$$I = I_{max}e^{-t/\tau}. \qquad [LR \text{ circuit without emf}]$$

The time constant τ is the time for the current to decrease to 37% of the original value, and again equals L/R.

These graphs show that there is always some "reaction time" when an electromagnet, for example, is turned on or off. We also see that an LR circuit has properties similar to an RC circuit (Section 19–6). Unlike the capacitor case, however, the time constant here is *inversely* proportional to R.