and the total emf is

$$\mathscr{E} = 2NBlv \sin\theta,$$

where we have multiplied by N, the number of loops in the coil.

If the coil is rotating with constant angular velocity ω, then the angle $\theta = \omega t$. We also have from the angular equations (Eq. 8–4) that $v = \omega r = \omega(h/2)$, where h is the length of bc or ad. Thus $\mathscr{E} = 2NB\omega l(h/2)\sin\omega t$, or

$$\mathscr{E} = NB\omega A \sin\omega t, \quad \textbf{(21–5)}$$

where $A = lh$ is the area of the loop. This equation holds for any shape coil, not just for a rectangle as derived. Thus, the output emf of the generator is sinusoidally alternating (see Fig. 21–18 and Section 18–7). Since ω is expressed in radians per second, we can write $\omega = 2\pi f$, where f is the frequency.

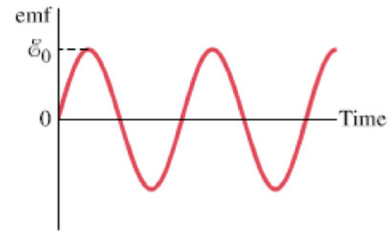


FIGURE 21–18 An ac generator produces an alternating current. The output emf $\mathscr{E} = \mathscr{E}_0 \sin\omega t$, where $\mathscr{E}_0 = NA\omega B$ (Eq. 21–5).

* 21–6 Back EMF and Counter Torque; Eddy Currents

* Back EMF

A motor turns and produces mechanical energy when a current is made to flow in it. From our description in Section 20–10 of a simple dc motor, you might expect that the armature would accelerate indefinitely due to the torque on it. However, as the armature of the motor turns, the magnetic flux through the coil changes and an emf is generated. This induced emf acts to oppose the motion (Lenz's law) and is called the **back emf** or **counter emf**. The greater the speed of the motor, the greater the back emf. A motor normally turns and does work on something, but if there were no load, the motor's speed would increase until the back emf equaled the input voltage. When there is a mechanical load, the speed of the motor may be limited also by the load. The back emf will then be less than the external applied voltage. The greater the mechanical load, the slower the motor rotates and the lower is the back emf ($\mathscr{E} \propto \omega$, Eq. 21–5).

EXAMPLE 21–9 **Back emf in a motor.** The armature windings of a dc motor have a resistance of 5.0 Ω. The motor is connected to a 120-V line, and when the motor reaches full speed against its normal load, the back emf is 108 V. Calculate (*a*) the current into the motor when it is just starting up, and (*b*) the current when the motor reaches full speed.

APPROACH As the motor is just starting up, it is turning very slowly, so there is no induced back emf. The only voltage is the 120-V line. The current is given by Ohm's law with $R = 5.0\,\Omega$. At full speed, we must include as emfs both the 120-V applied emf and the opposing back emf.

SOLUTION (*a*) At start up, the current is controlled by the 120 V applied to the coil's 5.0-Ω resistance. By Ohm's law,

$$I = \frac{V}{R} = \frac{120\,\text{V}}{5.0\,\Omega} = 24\,\text{A}.$$

(*b*) When the motor is at full speed, the back emf must be included in the equivalent circuit shown in Fig. 21–19. In this case, Ohm's law (or Kirchhoff's rule) gives

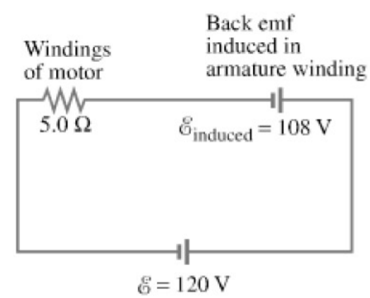
$$120\,\text{V} - 108\,\text{V} = I(5.0\,\Omega).$$

Therefore

$$I = \frac{12\,\text{V}}{5.0\,\Omega} = 2.4\,\text{A}.$$

NOTE This result shows that the current can be very high when a motor first starts up. This is why the lights in your house may dim when the motor of the refrigerator (or other large motor) starts up. The large initial current causes the voltage at the outlets to drop, since the house wiring has resistance and there is some voltage drop across it when large currents are drawn.

FIGURE 21–19 Circuit of a motor showing induced back emf. Example 21–9.



Effect of back emf on current