21–2 Faraday's Law of Induction; Lenz's Law

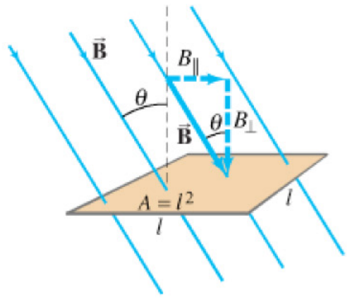


FIGURE 21–3 Determining the flux through a flat loop of wire. This loop is square, of side l and area $A = l^2$.

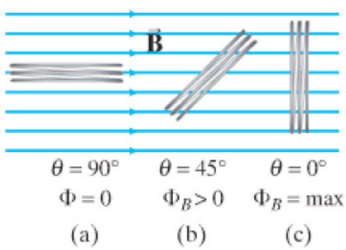
Faraday investigated quantitatively what factors influence the magnitude of the emf induced. He found first of all that the more rapidly the magnetic field changes, the greater the induced emf in a loop of wire. But the emf is not simply proportional to the rate of change of the magnetic field, $\vec{\mathbf{B}}$; it depends also on the loop's area and angle. That is, the emf is proportional to the rate of change of the **magnetic flux**, Φ_B, through the loop. Magnetic flux for a uniform magnetic field through a loop of area A is defined as

$$\Phi_B = B_\perp A = BA\cos\theta. \qquad [B \text{ uniform}] \quad \textbf{(21–1)}$$

Here $B_\perp$ is the component of the magnetic field $\vec{\mathbf{B}}$ perpendicular to the face of the loop, and θ is the angle between $\vec{\mathbf{B}}$ and a line perpendicular to the face of the loop. These quantities are shown in Fig. 21–3 for a square loop of side l whose area is $A = l^2$. When the face of the loop is parallel to $\vec{\mathbf{B}}$, $\theta = 90°$ and $\Phi_B = 0$. When $\vec{\mathbf{B}}$ is perpendicular to the loop, $\theta = 0°$, and

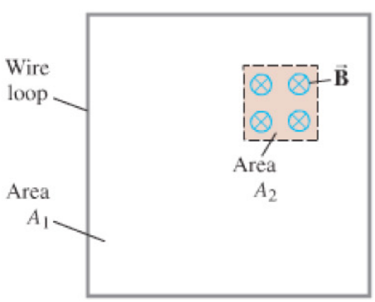
$$\Phi_B = BA. \qquad [\text{uniform } \vec{\mathbf{B}} \perp \text{loop face}]$$

FIGURE 21–4 Magnetic flux Φ_B is proportional to the number of lines of $\vec{\mathbf{B}}$ that pass through the loop.



As we saw in Chapter 20, the lines of $\vec{\mathbf{B}}$ (like lines of $\vec{\mathbf{E}}$) can be drawn such that the number of lines per unit area is proportional to the field strength. Then the flux Φ_B can be thought of as being proportional to the *total number of lines passing through the area enclosed by the loop*. This is illustrated in Fig. 21–4, where the loop is viewed from the side (on edge). For $\theta = 90°$, no magnetic field lines pass through the loop and $\Phi_B = 0$, whereas Φ_B is a maximum when $\theta = 0°$. The unit of magnetic flux is the tesla-meter^2; this is called a **weber**: $1\text{ Wb} = 1\text{ T}\cdot\text{m}^2$.

FIGURE 21–5 Example 21–1.



CONCEPTUAL EXAMPLE 21–1 **Determining flux.** A square loop of wire encloses area A_1 as shown in Fig. 21–5. A uniform magnetic field $\vec{\mathbf{B}}$ perpendicular to the loop extends over the area A_2. What is the magnetic flux through the loop A_1?

RESPONSE We assume that the magnetic field is zero outside the area A_2. The total magnetic flux through area A_1 is the flux through area A_2, which by Eq. 21–1 for a uniform field is BA_2, plus the flux through the remaining area $(= A_1 - A_2)$, which is zero because $B = 0$. So the total flux is $\Phi_B = BA_2 + 0(A_1 - A_2) = BA_2$. It is *not* equal to BA_1 because $\vec{\mathbf{B}}$ is not uniform over A_1.

EXAMPLE 21–2 **Calculate the flux.** A square loop of wire 10.0 cm on a side is in a 1.25-T magnetic field B. What are the maximum and minimum values of flux that can pass through the loop?

APPROACH The flux is given by Eq. 21–1. It is a maximum for $\theta = 0°$, which occurs when the plane of the loop is perpendicular to $\vec{\mathbf{B}}$. The minimum value occurs when $\theta = 90°$ and the plane of the loop is aligned with $\vec{\mathbf{B}}$.

SOLUTION From Eq. 21–1, the maximum value is

$$\Phi_B = BA\cos\theta = (1.25\text{ T})(0.100\text{ m})(0.100\text{ m})\cos 0° = 0.0125\text{ Wb}.$$

The minimum value is 0 Wb when $\theta = 90°$ and $\cos 90° = 0$.

EXERCISE A Find the flux in Example 21–2 when the perpendicular to the coil makes a 35° angle with $\vec{\mathbf{B}}$.

With our definition of flux, Eq. 21–1, we can now write down the results of Faraday's investigations. If the flux through a loop of wire changes by an amount $\Delta\Phi_B$ over a very brief time interval Δt, the induced emf at this instant is

FARADAY'S LAW OF INDUCTION

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t}. \qquad [1 \text{ loop}] \quad \textbf{(21–2a)}$$

This fundamental result is known as **Faraday's law of induction**, and it is one of the basic laws of electromagnetism.